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PHYSICS - Optional

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Quantum Mechanics 2015 - 2019

UPSC – PHYSICS Optional – 2015 Questions

1. Obtain an expression for the probability current for the plane wave $\varphi(x,t) = \exp[i(kx - \omega t]$. Interpret your result. **[10M]**

2. Using dimensional analysis, explain why the angular momentum of a particle cannot be λ^2.

[10M]

3. (i) Establish that :

hc = 1240 eV.nm

= 1240 KeV.fm

(ii) The energy levels of a hydrogen atom are give by $E_n = \left(\frac{-1}{n^2}\right) R_{yd}\ \ 1R_{yd} = hcR$.

Show that $R = 1.097 \times 10^7 m^{-1}$. **[10M]**

4. Solve the Schrodinger equation for a particle in a three dimensional rectangular potential barrier. Explain the terms degenerate and non-degenerate states in this context. **[30M]**

5. Write the time independent Schrodinger equation for a bouncing ball. **[10M]**

6. Normalized wave function of a particle is given below

$\varphi = Nexp\ \left(\frac{-x^2}{2a^2} + ikx\right)$ Find the expectation value of position. **[10M]**

7. A particle trapped in an infinitely deep square well of width a has a wave function

$\varphi = \left(\frac{2}{\pi}\right)^{1/2} \sin\left(\frac{\pi x}{a}\right)$. The walls are suddenly separated by infinite distance. Find the probability of the particle having momentum between p and p + dp. **[10M]**

UPSC – PHYSICS Optional – 2016 Questions

1. Find the energy, momentum and wavelength of a photon emitted by a hydrogen atom making a direct transition from an excited state with $n = 10$ to the ground state. Also find the recoil speed of the hydrogen atom in this process. **[10M]**

2. An electron is confined to move between two rigid walls separated by 10^{-9}m. Compute the de Broglie wavelengths representing the first three allowed energy states of the electron and the corresponding energies. **[10M]**

3. A typical atomic radius is about 5×10^{-15} m and the energy of β −particle emitted from a nucleus is at most of the order of 1 MeV. Prove on the basis of uncertainty principle that the electrons are not present in nuclei. **[10M]**

4. Using uncertainty principle, calculate the size and energy of the ground state of hydrogen atom. [**10M]**

5. Solve the Schrodinger equation for a step potential and calculate the transmission and reflection coefficient for the case when the kinetic energy of the particle E_0 is greater than the potential energy V (i.e., $E_0 > V$). [**20M]**

6. Write down the matrix representation of the three Pauli matrices σ_x, σ_y and σ_z. Prove that these matrices satisfy the following identities:

(i) $[\sigma_x, \sigma_y] = 2i\, \sigma_z$

(ii) $[\sigma^2 . \sigma_x] = 0$

(iii) $(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B}) = \vec{A}.\vec{B} + i\vec{\sigma}.(\vec{A} \times \vec{B})$

If $\vec{A}$ and $\vec{B}$ commute with Pauli matrices. [**20M]**

7. Calculate the density of states for an electron moving freely inside a metal with the help of quantum mechanical Schrodinger's equation for free particle in a box. [**10M]**

UPSC – PHYSICS Optional – 2017 Questions

1. A beam of 4.0 keV electrons from a source is incident on a target 50.0 cm away. Find the radius of the electron beamspot due to Heisenberg's uncertainty principle. [**10M]**

2. Calculate the lowest energy of an electron confined to move in a 1-dimensional potential well of width 10 nm. [**10M]**

3. Evaluate the most probable distance of the electron from nucleus of a hydrogen atom in its 2p state. What is the probability of finding the electron at this distance ? [**20M]**

4. Using Schrodinger equation, obtain the eigenfunctions and eigenvalues of energy for a 1-dimensional harmonic oscillator. Sketch the profiles of eigenfunctions for first three energy states. [**20M]**

5. Calculate the probability of transmission of an electron of 1.0 eV energy through a potential barrier of 4.0 eV and 0.1 nm width. [**10M]**

6. Explain why the square of the angular momentum (L^2) and only one of the components (L_x, L_y, L_z) $of\ L$ are regarded as constants of motion. [**15M]**

UPSC – PHYSICS Optional – 2018 Questions

1. The wave function of a particle is given as $\varphi(x) = \frac{1}{\sqrt{a}} e^{-|x|/a}$. Find the probability of locating the particle in the range $-a \leq x \leq a$. [**10M**]

2. Calculate the zero-point energy of a system consisting of a mass of 10^{-3} kg connected to a fixed point by a spring which is stretched by 10^{-2} m by a force of 10^{-1}N. The system is constrained to move only in one direction. [**10M**]

3. The general wave functions of harmonic oscillator (one – dimensional) are of the form

$$\mathrm{u}_n(x) = \sum_{k=0}^{\infty} a_k y^k e^{-y^2/2}$$

with $y = \sqrt{\frac{m\omega}{n}}\, x$, and coefficients a_k are determined by recurrence relations

$$a_{k+2} = \frac{2(k-n)}{(k+1)(k+2)} a_k$$

Corresponding energy levels are $E_n = \left(n + \frac{1}{2}\right) n\varphi$. Discuss the parity of these wave functions. What happens, if the potential for $x \leq 0$ is infinite (half harmonic oscillator)?

[**10M**]

4. Calculate the radius of electron orbit for Li^{++} in ground state. [**10M**]

5. Prove the following identities :

(i) $\left[\hat{p}_x, \hat{L}_y\right] = in\hat{p}_z$

(ii) $e^{i(\hat{\sigma}.\hat{n})} = \cos\theta + i(\hat{\sigma}.\hat{n}) \sin\theta$ [**15M**]

6. Which of the following functions is/are acceptable solution(s) of the Schrodinger equation?

(i) $\varphi(x) = Ae^{-ikx} + Be^{ikx}$

(ii) $\varphi\,(x) = Ae^{-kx} + Be^{kx}$

(iii) $\varphi(x) = A\sin 3kx + B\cos 5kx$

(iv) $\varphi(x) = A\sin 3kx + B\sin 5kx$

(v) $\varphi(x) = Atankx$ Explain your answer. [**15M**]

7. A bearn of particles of energy 9 eV is incident on a potential step 8 eV high from the left. What percentage of particles will reflect back? [**15M**]

8. Show that for free electron gas, the density of states in three dimensions (3D) varies as $E^{1/2}$, and this dependence changes to E^0 for 2D (quantum well), $E^{-1/2}$ for 1D (quantum wire) and δ function for OD (quantum dot). **[15M]**

UPSC – PHYSICS Optional – 2019 Questions

1. Show that the mass and linear momentum of a quantum mechanical particle can be given by $m = h/(\lambda v)$ and $p = h/\lambda$, respectively, where $h, \lambda\ and\ v$ are Planck's constant, wavelength and velocity of the particle, respectively. Comment on the wave-particle duality from these relations. **[10M]**

2. State and express mathematically the three uncertainty principles of Heisenberg. Highlight the physical significance of these principles in the development of Quantum Mechanics.

[10M]

3. For a free quantum mechanical particle under the influence of a one-dimensional potential, show that the energy is quantized in discrete fashion. How do these energy values differ from those of a linear harmonic oscillator? **[10M]**

4. How do you define density of states? Show that the density of states with wave vector less than $\vec{k}$ in a three-dimensional cubic box of volume V can be given by

$$D(\omega) = \frac{V}{2\pi^2} k^2 \left(\frac{dk}{d\omega}\right)$$

In the frequency spectrum between $\omega\ and\ \omega + d\omega$. Here, assume that the number of modes per unit range of k is L /(2π), L being the length of each side of the cubic box.

[20M]

5. Define Pauli spin matrices $\sigma_x, \sigma_y,\ and\ \sigma_z$. Using these definitions, prove the following :

(i) ${\sigma^2}_x = {\sigma^2}_y = {\sigma^2}_z = 1$

(ii) $\sigma_x\sigma_y = i\sigma_z;\ \sigma_z\sigma_x = i\sigma_y; \sigma_y\sigma_z = i\sigma_x$ **[15M]**

6. Define angular momentum of a particle and find out the three components of the angular momentum operator $\hat{L}^2 = -h^2 \left[r^2\Delta^2 - \frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)\right]$

Prove that the operator $\hat{L}^2$ can also be expressed as

$$\hat{L}^2 = -h^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta\ \frac{\partial}{\partial\theta}\right) + \frac{1}{Sin^2\theta} \frac{\partial^2}{\partial\phi^2}\right]$$

In spherical polar coordinates $(r, \theta, \emptyset)$. **[20M]**

7. Write down the Hamiltonian operator for a liner harmonic oscillator. Show that the energy eigenvalue of the same can be given by $E_n = \left(n + \frac{1}{2}\right) h\omega_0$ at energy state n with ω_0 being the natural frequency of vibration of the linear Gaussian form. [**15M]**