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Mathematics-Optional

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Numerical Methods & Computer Programming 2013 - 2021

UPSC – MATHEMATICS optional – 2013 Questions

1. In an examination, the number of students who obtained marks between certatin limits were given in the following table:

Marks	$30-40$	$40-50$	$50-60$	$60-70$	$70-80$
No. of Students	31	42	51	35	31

Using Newton forward interpolation formula, find the number of students whose marks lie between 45 and 50. **[10M]**

2. Develop an algorithm for Newton – Raphson mehod to solve $f(x)=0$ starting with initial iterate x_0, n be the number of iterations allowed, cps be the prescribed relative error and delta be the prescribed lower bound for $f'(x)$. **[20M]**

3. Use Euler's method with step size $h=0.15$ to compute the approximate value of $y(0.6)$, correct up to five decimal places from the initial value problem

$$y'=x(y+x)-2$$

$$y(0)=2$$ **[15M]**

4. The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in $\mathrm{km/hour}$.

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using composite Simpson's $\frac{1}{3}$ rule. **[15M]**

UPSC – MATHEMATICS optional – 2014 Questions

1. Find the solution of system

$$10x_1-2x_2-x_3-x_4=3$$

$$-2x_1+10x_2-x_3-x_4=15$$

$$-x_1-x_2+10x_3-2x_4=27$$

$$-x_1-x_2-2x_3+10x_4=-9$$

Using Gauss-Seidel method (make four iterations). **[15M]**

2. Solve the system of equations **[15M]**

$$2x_1 - x_2 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$-x_2 + 2x_3 = 1$$ Using Gauss-Seidel iteration method (Perform three iterations)

2. Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^x = 0$ correct up to four decimal places. **[10M]**

3. Use five subintervals to integrate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule. **[10M]**

4. Use only AND and OR logic gates to construct a logic circuit for the Boolean expression

$z = xy + uv$. **[10M]**

5. Use Runge-Kutta formula of fourth order to find the value of y at $x = 0.8$, where $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$. Take the step length $h = 0.2$. **[20M]**

6. Draw a flowchart for Simpson's one-third rule. **[15M]**

7. For any Boolean variables x and y, show that $x + xy = x$. **[15M]**

UPSC – MATHEMATICS optional – 2015 Questions

1. Find the principal (or canonical) disjunctive normal form in three variables p, q, r for the Boolean expression $((p \wedge q) \to r) \vee ((p \wedge q) \to -r)$. Is the given Boolean expression a contradiction or a tautology? **[10M]**

2. Find the Lagrange interpolating polynomial that fits the following data: **[20M]**

x	:	-1	2	3	4
$f(x)$	:	-1	11	31	69

Find $f(1.5)$

3. Solve the initial value problem $\frac{dy}{dx} = x(y - x)$, $y(2) = 3$ in the interval $[2, 2.4]$ using the Runge-Kutta fourth-order method with step size $h = 0.2$. **[15M]**

UPSC – MATHEMATICS optional – 2016 Questions

1. Convert the following decimal numbers to equivalent binary and hexadecimal numbers:

(i) 4096 (ii) 0.4375 (iii) 2048.0625 **[10M]**

2. Let $f(x) = e^{2x}\cos 3x$, for $x \in [0,1]$. Estimate the value of $f(0.5)$ using Lagrange interpolating polynomial of degree 3 over the nodes $x = 0, x = 0.3, x = 0.6$ and $x = 1$. Also, compute the error bound over the interval $[0,1]$ and the actual error $E(0.5)$. **[20M]**

3. For an integral $\int_{-1}^{1} f(x)dx$, show that the two-point Gauss quadrature rule is given by $\int_{-1}^{1} f(x)dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)$, Using this rule, estimate $\int_{2}^{4} 2xe^{x}dx$. **[15M]**

4. Let A, B, C be Boolean variables, $\overline{A}$ denote complement of A, $A + B$ is an expression for A OR B and $A \,.\, B$ is an expression for A AND B. Then simplify the following expression and draw a block diagram of the simplified expression and draw a block diagram of the simplified expression, using AND and OR gates.

$A \,.\, (A + B + C).\, (\overline{A} + B + C).\, (A + \overline{B} + C).\, (A + B + \overline{C})$. **[15M]**

UPSC – MATHEMATICS optional – 2017 Questions

1. Explain the main steps of the Gauss-Jordan method and apply this method to find the inverse of the matrix

$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$$ **[10M]**

2. Write the Boolean expression $z(y + z)(x + y + z)$ in its simplest form using Boolean postulate rules. Mention the rules used during simplification. Verify your result by constructing the truth table for the given expression and for its simplest form. **[10M]**

3. For given equidistant values u_{-1}, u_0, u_1 and u_2, a value is interpolated by Lagrange's formula. Show that it may be written in the form

$u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_0$, where $x + y = 1$. **[15M]**

4. Derive the formula

$$\int_a^b y\,dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) + 2(y_3 + y_6 + \cdots + y_{n-3})]$$

Is there any restriction on n? State that condition. What is the error bound in the case of Simpson's $\frac{3}{8}$ rule? **[20M]**

5. Write an algorithm in the form of a flow chart for Newton-Raphson method. Describe the cases of failure of this method. **[15M]**

UPSC – MATHEMATICS optional – 2018 Questions

1. Using Newton's forward difference formula find the lowest degree polynomial u_x when it is given that $u_1 = 1, u_2 = 9, u_3 = 25, u_4 = 55 \; and \; u_5 = 105$. **[10M]**

2.

Time (minutes)	2	4	6	8	10	12	14	16	18	20
Speed (Km/h)	10	18	25	29	32	20	11	5	2	8.5

Starting from rest in the beginning, the speed $(in\, Km/h)$ of a train at different times (in minutes) is given by the above table:

Using Simpson's $\frac{1}{3}rd$ rule, find the approximate distance travelled $(in\, Km)$ in 20 minutes from the beginning. **[10M]**

3. Find the equivalent of numbers given in a specified number system to the system mentioned against them

(i) $(111011.101)_2$ to decimal system

(ii) $(1000111110000.00101100)_2$ to hexadecimal system

(iii) $(C4F2)_{16}$ to decimal system

(iv) $(418)_{10}$ to binary system **[15M]**

4. Find the values of the constants a, b, c such that the quadrature formula $\int_0^h f(x)dx = h\left[af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right]$ is exact for polynomials of as high degree as possible, and hence find the order of the truncation error. **[15M]**

5. Simplify the Boolean expression: $(a + b) \cdot (\bar{b} + c) + b \cdot (\bar{a} + \bar{c})$ by using the laws of Boolean algebra. From its truth table write it in minterm normal form. **[15M]**

UPSC – MATHEMATICS optional – 2019 Questions

1. Using differentials, find an approximate value of $f(4.1, 4.9)$ where $f(x, y) = (x^3 + x^2 y)^{\frac{1}{2}}$. **[15M]**

2. Apply Newton-Raphson method, to find a real root of transcendental equation $x\log_{10} x = 1.2$, correct to three decimal places. **[10M]**

3. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$. Use four decimal places for calculation and step length 0.2. **[10M]**

4. Draw a flow chart and write a basic algorithm (in FORTRAN/C/C$^+$) for evaluating $y = \int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule. **[10M]**

5. Find the equivalent numbers given in a specified number to the system mentioned against them:

(i) Integer 524 in binary system.

(ii) 101010110101.101101011 to octal system.

(iii) decimal number 5280 to hexadecimal system.

(iv) Find the unknown number $(1101.101)_8 \rightarrow (?)10$. **[15M]**

6. Given the Boolean expression $X = AB + ABC + A\bar{B}\bar{C} + A\bar{C}$

(i) Draw the logical diagram for the expression.

(ii) Minimize the expression

(iii) Draw the logical diagram for the reduced expression. **[15M]**

UPSC – MATHEMATICS optional – 2020 Questions

1. Show that the equation: $f(x) = \cos\frac{\pi(x+1)}{8} + 0.148x - 0.9062 = 0$ has one root in the interval $(-1, 0)$ and one in $(0, 1)$. Calculate the negative root correct to four decimal places using Newton-Raphson method. **[10M]**

2. Let $g(w, x, y, z) = (w + x + y)(x + \bar{y} + z)(w + \bar{y})$ be a Boolean function. Obtain the conjunctive normal form for $g(w, x, y, z)$. Also express $g(w, x, y, z)$ as a product of maxterms.

[10M]

3. For the solution of the system of equations:
$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

Set up the Gauss-Seidel iterative scheme and iterate three times starting with the initial vector $X^{(0)} = 0$. Also find the exact solutions and compare with the iterated solutions. **[15M]**

4. Find a quadrature formula $\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$ which is exact for polynomials of highest possible degree. Then use the formula to evaluate $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ (correct up to three decimal places). **[20M]**

5. Write the three point Lagrangian interpolating polynomial relative to the points $x_0, x_0 + \varepsilon$ and x_1. Then by taking the limit $\varepsilon \to 0$, establish the relation **[15M]**

$$f(x) = \frac{(x_1 - x)(x + x_1 - 2x_0)}{(x_1 - x_0)^2} f(x_0) + \frac{(x - x_0)(x_1 - x)}{(x_1 - x_0)} f'(x_0) + \frac{(x - x_0)^2}{(x_1 - x_0)} f(x_1) + E(x)$$

Where $E(x) = \frac{1}{6}(x - x_0)^2(x - x_1)f'''(\xi)$

is the error function and $\min.(x_0, x_0 + \varepsilon, x_1) < \xi \max.(x_0, x_0 + \varepsilon, x_1)$

6. Given the Boolean expression $X = AB + ABC + A\bar{B}\bar{C} + A\bar{C}$

(i) Draw the logical diagram for the expression.

(ii) Minimize the expression

(iii) Draw the logical diagram for the reduced expression. **[15M]**

UPSC – MATHEMATICS optional – 2021 Questions

1. Find a positive root of the equation $3x = 1 + cosx$ by a numerical technique using initial values $0, \frac{\pi}{2}$; and further improve the result using Newton-Raphson method correct to 8 significant figures. **[10M]**

2. Convert $(3798 \cdot 3875)_{10}$ into octal and hexadecimal equivalents. **[05M]**

3. Obtain the principal conjunctive normal form of $(\neg P \to R) \wedge (Q \leftrightarrows P)$. **[05M]**

4. Obtain the Boolean function $F(x, y, z)$ based on the table given below. Then simplify $F(x, y, z)$ and draw the corresponding GATE network: **[15M]**

x	y	z	$F(x, y, z)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

5. Solve the system of equations

$$3x_1 + 9x_2 - 2x_3 = 11$$
$$4x_1 + 2x_2 + 13x_3 = 24$$
$$4x_1 - 2x_2 + x_3 = -8$$

Correct up to 4 significant figures by using Gauss-Seidel method after verifying whether the method is applicable in your transformed form of the system. **[15M]**

6. Derive Newton's backward difference interpolation formula and also do error analysis. **[15M]**