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Mathematics-Optional

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UPSC – MATHEMATICS optional – 2013 Questions

1. Form a partial differential equation by eliminating the arbitrary functions f and g from $z = yf(x) + xg(y)$. **[10M]**

2. Reduce the equation

$$y\frac{\partial^2 z}{\partial x^2} + (x+y)\frac{\partial^2 z}{\partial x \partial y} + x\frac{\partial^2 z}{\partial y^2} = 0$$

To its canonical form when $x \neq y$. **[10M]**

3. Solve $(D^2 + DD' - 6D'^2)z = x^2 \sin(x+y)$

Where D and D′ denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$. **[15M]**

4. Find the surface which intersects the surfaces of the system

$$z(x+y) = \mathrm{C}(3z+1), (\mathrm{C} \text{ being a constant})$$

orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. **[15M]**

5. A tightly stretched string with fixed end points $x = 0$ *and* $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity $\lambda . x(l-x)$, find the displacement of the string at any distance x from one end at any time t. **[20M]**

6. Two equal rods AB and BC, each of length l, smoothly jointed at B, are suspended from A and oscillate in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$ where $n^2 = \left(3 + \frac{6}{\sqrt{7}}\right)\frac{g}{l}$. **[15M]**

UPSC – MATHEMATICS optional – 2014 Questions

1.Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)z = 24(y-x)$. **[10M]**

2. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. **[15M]**

3. Find the deflection of a vibrating string (length $= \pi$, ends fixed, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$) corresponding to zero initial velocity and initial deflection

$$f(x) = k(\sin x - \sin 2x)$$ **[15M]**

4. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1, t > 0$, given that

(i) $u(x,0) = 0,\ 0 \leq x \leq 1$

(ii) $\frac{\partial u}{\partial t}(x,0) = x^2,\ 0 \leq x \leq 1$

(iii) $u(0,t) = u(1,t) = 0$, for all t **[15M]**

UPSC – MATHEMATICS optional – 2015 Questions

1. Solve the partial differential equation

$$(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. **[10M]**

2. Solve $(D^2 + DD' - 2D'^2)u = e^{x+y}$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. **[10M]**

3. Solve for the general solution $p\cos(x+y) + q\sin(x+y) = z$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. **[15M]**

4. Find the solution of the initial-boundary value problem.

$$u_t - u_{xx} + u = 0, \quad 0 < x < l, t > 0$$

$$u(0,t) = u(l,t) = 0, \quad t \geq 0$$

$$u(x,0) = x(l-x), \quad 0 < x < l$$

[15M]

5. Reduce the second-order partial differential equation

$$x^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

Into canonical form. Hence, find its general solution. **[15M]**

UPSC – MATHEMATICS optional – 2016 Questions

1. Find the general equation of surfaces orthogonal to the family of spheres given by $x^2 + y^2 + z^2 = cz$. **[10M]**

2. Find the general integral of the partial differential equation

$$(y + zx)p - (x + yz)q = x^2 - y^2.$$

[10M]

3. Determine the characteristics of the equation $z = p^2 - q^2$, and find the integral surface which passes through the parabola $4z + x^2 = 0, y = 0$. **[15M]**

4. Solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$ **[15M]**

5. Find the temperature $u(x,t)$ in a bar of silver of length 10 cm and constant cross-section of area 1 cm^2. Let density $\rho = 10.6\, g/cm^2$, thermal conductivity $K = 1.04\, cal/(cm\,\sec{}^\circ C)$ and specific heat $\sigma = 0.056$ cal/g °C. The bar is perfectly isolated laterally, with ends kept at 0°C and initial temperature $f(x) = \sin(0.1\,\pi x)$°C. Note that $u(x,t)$ follows the heat equation $u_t = c^2 u_{xx}$, where $c^2 = K/(\rho\,\sigma)$. **[20M]**

UPSC – MATHEMATICS optional – 2017 Questions

1. Let $f = u + iv$ be an analytic function on the unit disc $D = \{z \in C : |z| < 1\}$. Show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$$

at all points of D. **[15M]**

2. Solve $(D^2 - 2DD' + D'^2)\, z = e^{x+2y} + x^3 + \sin 2x,$ **[10M]**

Where $D \equiv \frac{\partial}{\partial x}, \quad D' = \frac{\partial}{\partial y}, \; D^2 = \frac{\partial^2}{\partial x^2}, D'^2 = \frac{\partial^2}{\partial y^2}.$

3. Let Γ be a closed curve in xy-plane and let S denote the region bounded by the curve Γ.

Let $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x,y) \forall (x,y) \in S$. If f is prescribed at each point $(x,y) of\ S\ and\ w$ is prescribed on the boundary Γof S, then prove that any solution $w = w(x,y)$, satisfying these conditions, is unique. **[10M]**

4. Find a complete integral of the partial differential equation
$2(pq + yp + qx) + x^2 + y^2 = 0.$ **[15M]**

5. Reduce the equation $y^2 \frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and hence solve it. **[15M]**

6. Given the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}; t > 0$, where $c^2 = \frac{T}{m}, T$ is the constant tension in the string and m is the mass per unit length of the string.

(i) Find the appropriate solution of the above wave equation.

(ii) Find also the solution under the conditions $y(0,t) = 0,\ y(l,t) = 0\, for\ all\ t$ and $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0, y(x,0) = a \sin\frac{\pi x}{l}, 0 < x < l. a > 0.$ **[20M]**

UPSC – MATHEMATICS optional – 2018 Questions

1. Find the partial differential equation of the family of all tangent planes to the ellipsoid : $x^2 + 4y^2 + 4z^2 = 4$, which are not perpendicular to the xy plane **[10M]**

2. Solve the partial differential equation:

$$(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$$

Where $D \equiv \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. **[15M]**

3. A thin annulus occupies the region $0 < a \leq r \leq b,\ o \leq \theta \leq 2\pi$. The faces are insulated. Along the inner edge the temperature is maintained at $0°$, while along the outer edge the temperature is held at $T = K\cos\frac{\theta}{2}$, where K is a constant. Determine the temperature distribution in the annulus. **[20M]**

4. Find the general solution of the partial differential equation: $(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$, where $p = \frac{\partial z}{\partial x}$,$q = \frac{\partial z}{\partial y}$, and find its integral surface that passes through the curve: $x = t, y = t^2, z = 1$. **[15M]**

UPSC – MATHEMATICS optional – 2019 Questions

1. Form a partial differential equation of the family of surfaces given by the following expression: $\psi(x^2 + y^2 + 2z^2,\ y^2 - 2zx) = 0$. **[10M]**

2. Solve the first order quasilinear partial differential equation by the method of characteristics: **[15M]**

$$x\frac{\partial u}{\partial x} + (u - x - y)\frac{\partial u}{\partial y} = x + 2y \text{ in } x > 0, -\infty < y < \infty \text{ with } u = 1 + y \text{ on } x = 1.$$

3. Reduce the following second order partial differential equation to canonical form and find the general solution:

$$\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} = \frac{\partial y}{\partial x} + 12x. \quad \textbf{[20M]}$$