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## Online IAS Academy

# Mathematics-Optional

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Linear Programming 2013-2019

# UPSC – MATHEMATICS optional – 2013 Questions

**1.** Maximize $z = 2x_1 + 3x_2 - 5x_3$

Subject to $x_1 + x_2 + x_3 = 7$

and $2x_1 - 5x_2 + x_3 \geq 10,\ x_i \geq 0.$ **[10M]**

**2.** Solve the minimum time assignment problem: **[15M]**

Machines

| Jobs | $M_1$ | $M_2$ | $M_3$ | $M_3$ | $M_4$ |
|---|---|---|---|---|---|
| | $J_1$ | 3 | 12 | 5 | 14 |
| | $J_2$ | 7 | 9 | 8 | 12 |
| | $J_3$ | 5 | 11 | 10 | 12 |
| | $J_4$ | 6 | 14 | 4 | 11 |

**3.** Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

subject to the constraints

$$x_1 + 2x_2 - 2x_3 + 4x_4 \leq 40$$

$$2x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$4x_1 - 2x_2 + x_3 - x_4 \leq 10$$

$$x_i \geq 0$$ **[20M]**

# UPSC – MATHEMATICS optional – 2014 Questions

**1.** Solve graphically:

Maximize $Z = 6x_1 + 5x_2$

Subject to $2x_1 + x_2 \leq 16$

$$x_1 + x_2 \leq 11$$

$$x_1 + 2x_2 \geq 6$$

$$5x_1 + 6x_2 \leq 90$$

$$x_1, x_2 \geq 0$$ **[10M]**

**2.** Find the initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also, find its optimal solution and the minimum transportation cost:

**[20M]**

Destinations

| Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $O_1$ | 6 | 4 | 1 | 5 | 14 |
| $O_2$ | 8 | 9 | 2 | 7 | 16 |
| $O_3$ | 4 | 3 | 6 | 2 | 5 |
| Demand | 6 | 10 | 15 | 4 | |

**3.** Find all optimal solutions of the following linear programming problem by the simplex method:

Maximize $Z = 30x_1 + 24x_2$

subject to $5x_1 + 4x_2 \leq 200$

$x_1 \leq 32$

$x_2 \leq 40$

$x_1, x_2 \geq 0$ **[20M]**

## UPSC – MATHEMATICS optional – 2015 Questions

**1.** Solve the following assignment problem to maximize the sales: **[10M]**

Territories

| Salesmen | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 3 | 4 | 5 | 6 | 7 |
| B | 4 | 15 | 13 | 7 | 6 |
| C | 6 | 13 | 12 | 5 | 11 |
| D | 7 | 12 | 15 | 8 | 5 |
| E | 8 | 13 | 10 | 6 | 9 |

**2.** Consider the following linear programming problem:

Maximize $Z = x_1 + 2x_2 - 3x_3 + 4x_4$

Subject to $x_1 + x_2 + 2x_3 + 3x_4 = 12$

$x_2 + 2x_3 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

(i) Using the definition, find its all basic solutions. Which of these are degenerate basic feasible solutions and which are non-degenerate basic feasible solution?

(ii) Without solving the problem, show that it has an optimal solution. Which of the basic feasible solution(s) is /are optimal? **[20M]**

**3.** Solve the following linear programming problem by the simplex method. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem :

Maximize $Z = 2x_1 - 4x_2 + 5x_3$

Subject to $x_1 + 4x_2 - 2x_3 \leq 2$

$$-x_1 + 2x_2 + 3x_3 \leq 1$$

$$x_1, x_2, x_3 \geq 0$$ **[20M]**

## UPSC – MATHEMATICS optional – 2016 Questions

**1.** Find the maximum value of $5x + 2y$ with constraints

$x + 2y \geq 1,\ 2x + y \leq 1,\ x \geq 0\ and\ y \geq 0$ by graphical method. **[10M]**

**2.** Maximize $z = 2x_1 + 3x_2 + 6x_3$

Subject to

$$2x_1 + x_2 + x_3 \leq 5$$

$$3x_2 + 2x_3 \leq 6$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Is the optimal solution unique? Justify your answer. **[20M]**

## UPSC – MATHEMATICS optional – 2017 Questions

**1.** Using graphical method, find the maximum value of $2x + y$

Subject to $4x + 3y \leq 12$

$$4x + y \leq 8$$

$$4x - y \leq 8$$

$$x, y \geq 0.$$ **[20M]**

**2.** Solve the following linear programming problem by simplex method: **[20M]**

Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0.$

**3.** Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method and find the cost. **[15M]**

Destinations

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | |
|---|---|---|---|---|---|---|
| $O_1$ | 4 | 7 | 0 | 3 | 6 | 14 |
| $O_2$ | 1 | 2 | -3 | 3 | 8 | 9 |
| $O_3$ | 3 | -1 | 4 | 0 | 5 | 17 |
| | 8 | 3 | 8 | 13 | 8 | |

Origins — Supply

Demand

## UPSC – MATHEMATICS optional – 2018 Questions

**1.** An agricultural firm has 180 tons of nitrogen fertilizer, 250 tons of phosphate and 220 tons of potash. It will be able to sell a mixture of these substances in their respectively ration 3 : 3 : 4 at a profit of Rs. 1500 per ton and a mixture in the ratio 2 : 4 : 2 at a profit of Rs. 1200 per ton. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. **[10M]**

**2.** Solve the following linear programming problem by Big M-method and show that the problem has finite optimal solutions. Also find the value of the objective function:

Maximize $z = 3x_1 + 5x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60,$

$x_1, x_2 \geq 0.$ **[20M]**

**3.** How many basic solutions are there in the following linearly independent set of equations? Find all of them.

$$2x_1 - x_2 + 3x_3 + x_4 = 6$$

$$4x_1 - 2x_2 - x_3 + 2x_4 = 10.$$ **[15M]**

**4.**

| | | Machine | | | | |
|---|---|---|---|---|---|---|
| | | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
| Operator | $O_1$ | 24 | 29 | 18 | 32 | 19 |
| | $O_2$ | 17 | 26 | 34 | 22 | 21 |
| | $O_3$ | 27 | 16 | 28 | 17 | 25 |
| | $O_4$ | 22 | 18 | 28 | 30 | 24 |
| | $O_5$ | 28 | 16 | 31 | 24 | 27 |

In a factory there are five operators $O_1, O_2, O_3, O_4, O_5$ and five machines $M_1, M_2, M_3, M_4, M_5$. The operating costs are given when the $O_i$ operator operates the $M_j$ machine$(i, j = 1,2, \ldots\ldots,5)$. But there is a restriction that $O_3$ cannot be allowed to operate the third machine $M_3$ and $O_2$ cannot be allowed to operate the fifth machine $M_5$.The cost matrix is given above. Find the optimal assignment and the optimal assignment cost also. **[15M]**

## UPSC – MATHEMATICS optional – 2019 Questions

**1.** Use graphical method to solve the linear programming problem.

Maximize $Z = 3x_1 + 2x_2$

Subject to

$$x_1 - x_2 \geq 1,$$

$$x_1 + x_3 \geq 3$$

and $x_1, x_2, x_3 \geq 0$ **[10M]**

**2.** Solve the linear programming problem using Simplex method.

Minimize $Z = x_1 + 2x_2 - 3x_3 - 2x_4$

Subject to

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$

$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$

and $x_1, x_2, x_3, x_4 \geq 0$ **[15M]**

**3.** Consider the following LPP,

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to

$$x_1 + x_2 + x_3 = 4$$

$$x_1 + 4x_2 + x_4 = 8$$

and $x_1, x_2, x_3, x_4 \geq 0$

Use the dual problem to verify that the basic solution $(x_1, x_2)$ is not optimal. **[10M]**