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Mathematics-Optional

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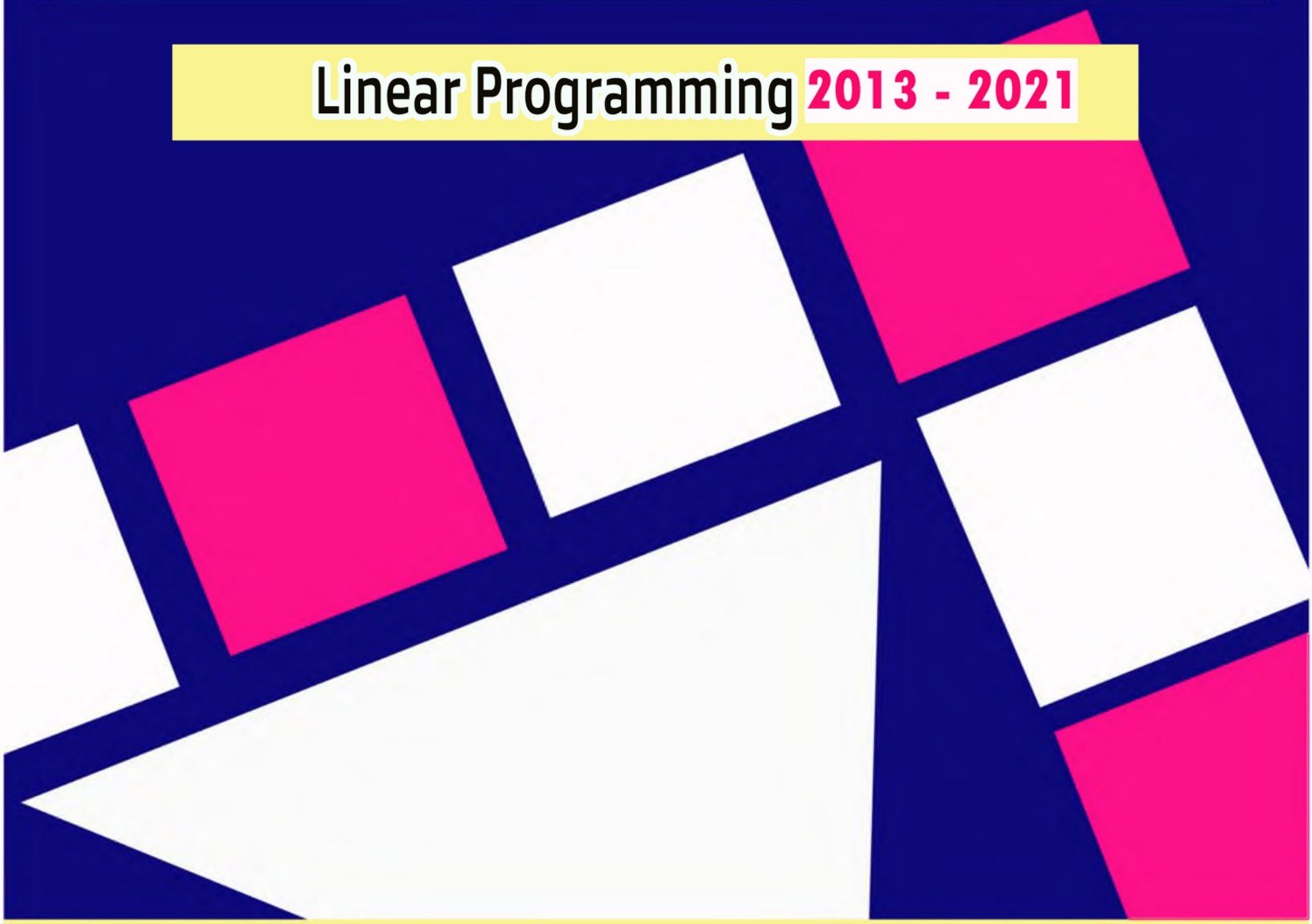
Linear Programming 2013 - 2021



UPSC – MATHEMATICS optional – 2013 Questions

1. Maximize $z = 2x_1 + 3x_2 - 5x_3$

Subject to $x_1 + x_2 + x_3 = 7$

and $2x_1 - 5x_2 + x_3 \geq 10,\ x_i \geq 0.$ **[10M]**

2. Solve the minimum time assignment problem: **[15M]**

Machines

Jobs	M_1	M_2	M_3	M_3	M_4
	J_1	3	12	5	14
	J_2	7	9	8	12
	J_3	5	11	10	12
	J_4	6	14	4	11

3. Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

subject to the constraints

$$x_1 + 2x_2 - 2x_3 + 4x_4 \leq 40$$

$$2x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$4x_1 - 2x_2 + x_3 - x_4 \leq 10$$

$$x_i \geq 0$$ **[20M]**

UPSC – MATHEMATICS optional – 2014 Questions

1. Solve graphically:

Maximize $Z = 6x_1 + 5x_2$

Subject to $2x_1 + x_2 \leq 16$

$$x_1 + x_2 \leq 11$$

$$x_1 + 2x_2 \geq 6$$

$$5x_1 + 6x_2 \leq 90$$

$$x_1, x_2 \geq 0$$ **[10M]**

2. Find the initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also, find its optimal solution and the minimum transportation cost:

[20M]

Destinations

Origins		D_1	D_2	D_3	D_4	Supply
	O_1	6	4	1	5	14
	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
	Demand	6	10	15	4	

3. Find all optimal solutions of the following linear programming problem by the simplex method:

Maximize $Z = 30x_1 + 24x_2$

subject to $5x_1 + 4x_2 \leq 200$

$x_1 \leq 32$

$x_2 \leq 40$

$x_1, x_2 \geq 0$ **[20M]**

UPSC – MATHEMATICS optional – 2015 Questions

1. Solve the following assignment problem to maximize the sales: **[10M]**

Territories

Salesmen	I	II	III	IV	V
A	3	4	5	6	7
B	4	15	13	7	6
C	6	13	12	5	11
D	7	12	15	8	5
E	8	13	10	6	9

2. Consider the following linear programming problem:

Maximize $Z = x_1 + 2x_2 - 3x_3 + 4x_4$

Subject to $x_1 + x_2 + 2x_3 + 3x_4 = 12$

$x_2 + 2x_3 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

(i) Using the definition, find its all basic solutions. Which of these are degenerate basic feasible solutions and which are non-degenerate basic feasible solution?

(ii) Without solving the problem, show that it has an optimal solution. Which of the basic feasible solution(s) is /are optimal? **[20M]**

3. Solve the following linear programming problem by the simplex method. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem:

Maximize $Z = 2x_1 - 4x_2 + 5x_3$

Subject to $x_1 + 4x_2 - 2x_3 \leq 2$

$-x_1 + 2x_2 + 3x_3 \leq 1$

$x_1, x_2, x_3 \geq 0$ **[20M]**

UPSC – MATHEMATICS optional – 2016 Questions

1. Find the maximum value of $5x + 2y$ with constraints

$x + 2y \geq 1,\ 2x + y \leq 1,\ x \geq 0\ and\ y \geq 0$ by graphical method. **[10M]**

2. Maximize $z = 2x_1 + 3x_2 + 6x_3$

Subject to

$2x_1 + x_2 + x_3 \leq 5$

$3x_2 + 2x_3 \leq 6$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

Is the optimal solution unique? Justify your answer. **[20M]**

UPSC – MATHEMATICS optional – 2017 Questions

1. Using graphical method, find the maximum value of $2x + y$

Subject to $4x + 3y \leq 12$

$4x + y \leq 8$

$4x - y \leq 8$

$x, y \geq 0.$ **[20M]**

2. Solve the following linear programming problem by simplex method: **[20M]**

Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0.$

3. Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method and find the cost. **[15M]**

		Destinations					
		D_1	D_2	D_3	D_4	D_5	
Origins	O_1	4	7	0	3	6	14
	O_2	1	2	-3	3	8	9
	O_3	3	-1	4	0	5	17
		8	3	8	13	8	
		Demand					

Supply

UPSC – MATHEMATICS optional – 2018 Questions

1. An agricultural firm has 180 tons of nitrogen fertilizer, 250 tons of phosphate and 220 tons of potash. It will be able to sell a mixture of these substances in their respectively ration 3 : 3 : 4 at a profit of Rs. 1500 per ton and a mixture in the ratio 2 : 4 : 2 at a profit of Rs. 1200 per ton. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. **[10M]**

2. Solve the following linear programming problem by Big M-method and show that the problem has finite optimal solutions. Also find the value of the objective function:

Maximize $z = 3x_1 + 5x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60,$

$x_1, x_2 \geq 0.$ **[20M]**

3. How many basic solutions are there in the following linearly independent set of equations? Find all of them.

$$2x_1 - x_2 + 3x_3 + x_4 = 6$$

$$4x_1 - 2x_2 - x_3 + 2x_4 = 10.$$ **[15M]**

4.

		Machine				
		M_1	M_2	M_3	M_4	M_5
Operator	O_1	24	29	18	32	19
	O_2	17	26	34	22	21
	O_3	27	16	28	17	25
	O_4	22	18	28	30	24
	O_5	28	16	31	24	27

In a factory there are five operators O_1, O_2, O_3, O_4, O_5 and five machines M_1, M_2, M_3, M_4, M_5. The operating costs are given when the O_i operator operates the M_j machine$(i, j = 1,2, \ldots\ldots,5)$. But there is a restriction that O_3 cannot be allowed to operate the third machine M_3 and O_2 cannot be allowed to operate the fifth machine M_5.The cost matrix is given above. Find the optimal assignment and the optimal assignment cost also. **[15M]**

UPSC – MATHEMATICS optional – 2019 Questions

1. Use graphical method to solve the linear programming problem.

Maximize $Z = 3x_1 + 2x_2$

Subject to

$$x_1 - x_2 \geq 1,$$

$$x_1 + x_3 \geq 3$$

and $x_1, x_2, x_3 \geq 0$ **[10M]**

2. Solve the linear programming problem using Simplex method.

Minimize $Z = x_1 + 2x_2 - 3x_3 - 2x_4$

Subject to

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$

$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$

and $x_1, x_2, x_3, x_4 \geq 0$ **[15M]**

3. Consider the following LPP,

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to

$$x_1 + x_2 + x_3 = 4$$

$$x_1 + 4x_2 + x_4 = 8$$

and $x_1, x_2, x_3, x_4 \geq 0$

Use the dual problem to verify that the basic solution (x_1, x_2) is not optimal. **[10M]**

UPSC – MATHEMATICS optional – 2020 Questions

1. UPSC maintenance section has purchased sufficient number of curtain cloth pieces to meet the curtain requirement of tis building. The length of each piece is 17 feet. The requirement according to curtain length is as follows:

Curtain length (in feet)	Number required
5	700
9	400
7	300

The width of all curtains is same as that of available pieces. Form a linear programming problem in standard form that decides the number of pieces cut in different ways so that the total trim loss is minimum. Also give a basic feasible solution to it. **[10M]**

2. Solve the linear programming problem using simplex method:

Minimize $z = -6x - 2x_2 - 5x_3$

Subject to $2x_1 - 3x_2 + x_3 \leq 14$

$-4x_1 + 4x_2 + 10x_3 \leq 46$

$2x_1 + 2x_2 - 4x_3 \leq 37$

$x_1 \geq 2,\ x_2 \geq 1,\ x_3 \geq 3$ **[15M]**

3.

	D_1	D_2	D_3	D_4	
S_1	10	0	20	11	15
S_2	12	8	9	20	25
S_3	0	14	16	18	10
	5	20	15	10	

Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method and use it to find the optimal solution and the transportation cost of the problem. **[20M]**

UPSC – MATHEMATICS optional – 2021 Questions

1. A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix. Assign all the jobs to these five employees to minimize the total processing time: **[10M]**

		Employees				
		I	*II*	*III*	*IV*	*V*
Jobs	*A*	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

2. Convert the following LPP into dual LPP:

Minimize $Z = x_1 - 3x_2 - 2x_3$

Subject to

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 - 4x_2 \geq 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

Where $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign. **[15M]**

3. Solve the following linear programming problem using Big M method:

Maximize $Z = 4x_1 + 5x_2 + 2x_3$

Subject to

$$5x_1 + x_2 + x_3 \geq 10$$

$$x_1 + 3x_2 + x_3 \leq 12$$

$$x_1 + x_2 + x_3 = 6$$

$$x_1, x_2, x_3 \geq 0$$ **[15M]**