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Mathematics-Optional

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REAL ANALYSIS 2013-2019

UPSC – MATHEMATICS optional – 2013 Questions

1. Let $f(x) = \begin{cases} \frac{x^2}{2} + 4 \, if \, x \geq 0 \\ -\frac{x^2}{2} + 2 \, if \, x < 0 \end{cases}$

Is f Riemann integrable in the interval [-1, 2] ? why ? Does there exist a function g such that $g'(x) = f(x)$? Justify your answer. **[10M]**

2. Show that the series $\sum_1^\infty \frac{(-1)^{n-1}}{n+x^2}$, is uniformly convergent but not absolutely for all real values of x. **[13M]**

3. Show that every open subset of R is a countable union of disjoint open intervals.

[10M]

4. Let $f(x,y) = y^2 + 4xy + 3x^2 + x^3 + 1$. At what points will $f(x,y)$ have a maximum or minimum ? **[10M]**

5.Let [X] denote the integer part of the real number x, i.e., if $n \leq x < n+1$ where n is an integer, then $[x] = n$. Is the function $f(x) = [x]^2 + 3$ Riemann integrable in [-1, 2] ? If not, explain why. If it is integrable, compute $\int_{-1}^{2}([x]^2 + 3)dx$. **[10M]**

6.Test the convergence of the improper integral $\int_1^\infty \frac{dx}{x^2(1+e^{-x})}$ **[10M]**

UPSC – MATHEMATICS optional – 2014 Questions

1. Integrate $\int_0^1 f(x)dx$, where $f(x) = \begin{cases} 2x \sin\frac{1}{x} - \cos\frac{1}{x}, & x \in [0,1] \\ 0, & x = 0 \end{cases}$ **[15M]**

2. Obtain $\frac{\partial^2 f(0,0)}{\partial x \partial y}$ and $\frac{\partial^2 f(0,0)}{\partial y \partial x}$ for the function $f(x,y) = \begin{cases} \frac{xy(3x^2-2y^2)}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$ Also discuss the continuity $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ at $(0,0)$ **[15M]**

2014

3. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$ by the method of Lagrange multipliers. **[15M]**

UPSC – MATHEMATICS optional – 2015 Questions

1.Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$.

[10M]

2.Is the function $f(x) = \begin{cases} \frac{1}{n}, & \frac{1}{n+1} < x \leq \frac{1}{n} \\ 0, & x = 0 \end{cases}$

Riemann Integrable ? If yes, obtain the value of $\int_0^1 f(x)dx$. **[15M]**

3. Test the series of functions $\sum_{n=1}^{\infty}\frac{nx}{1+n^2x^2}$ for uniform convergence. **[15M]**

4. Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + 3y^2 - y$ over the region $x^2 + 2y^2 \leq 1$. **[15M]**

UPSC – MATHEMATICS optional – 2016 Questions

1. For the function $f: (0, \infty) \to R$ given by

$f(x) = x^2 sin\frac{1}{x}, 0 < x < \infty,$ show that there is a differentiable function $g: R \to R$ that extends f. **[10M]**

2.Two sequences $\{x_n\} and \{y_n\}$ are defined inductively by the following :

$x_1 = \frac{1}{2}, \ y_1 = 1$ and $x_n - \sqrt{x_{n-1}\, y_{n-1}}, n = 2, 3, 4, ...$

$$\frac{1}{y_n} = \frac{1}{2}\left(\frac{1}{x_n} + \frac{1}{y_{n-1}}\right), \quad n = 2, 3, 4, ...$$

Prove that $x_n - 1 < x_n < y_n < y_{n-1}, n = 2, 3, 4, ...$

and deduce that both the sequences converge to the same limit *l*, where $\frac{1}{2} < l < 1$. **[10M]**

3.Show that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$ is conditionally convergent. (If you use any theorem(s) to show it, then you must give a proof of that theorem(s). **[15M]**

4.Find the relative maximum and minimum values of the function

$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2.$ **[15M]**

5.Let $f: R \to R$ be a continuous function such that $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ exist and are finite. Prove that f is uniformly continuous on R. **[15M]**

UPSC – MATHEMATICS optional – 2017 Questions

1.Let $x_1 = 2$ and $x_{n+1} = \sqrt{x_n + 20},\ n = 1, 2, 3, \ldots$ Show that the sequence $x_1, x_2, x_3, \ldots$ is convergent. **[10M]**

2.Find the supremum and the infimum of $\frac{x}{sinx}$ on the interval $(0, \frac{\pi}{2}]$. **[10M]**

3.Let $f(t) = \int_0^t [x]dx,$

Where [x] denotes the largest integer less than or equal to x.

(i) Determine all the real numbers t at which f is differentiable.

(ii) Determine all the real numbers t at which f is continuous but not differentiable. **[15M]**

4. For a function $f: C \to C$ and $n \geq 1$, let $f^{(n)}$ denote the nth derivative of f and $f^{(0)} = f$. Let f be an entire function such that for some $n \geq 1, f^{(n)}\left(\frac{1}{k}\right) = 0$ for all $k = 1, 2, 3, \ldots\ldots$ Show that f is a polynomial. **[15M]**

5. Let $\sum_{n=1}^{\infty} x_n$ be a conditionally convergent series of real numbers. Show that there is a rearrangement $\sum_{n=1}^{\infty} x_{\pi(n)}$ of the series $\sum_{n=1}^{\infty} x_n$ that converges to 100. **[20M]**

UPSC – MATHEMATICS optional – 2018 Questions

1.Prove the inequality : $\frac{\pi^2}{9} < \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx} dx < \frac{2\pi^2}{9}$. **[10M]**

2.Find the range of $p(> 0)$ for which the series :

$\frac{1}{(1+a)^P} - \frac{1}{(2+a)^P} + \frac{1}{(3+a)^P} - \cdots, a > 0$, is

(i) absolutely convergent and (ii) conditionally convergent. **[10M]**

3.Show that if a function f defined on an open interval (a, b) of IR is convex, then f is continuous. Show, by example, if the condition of open interval is dropped, then the convex function need not be continuous. **[15M]**

4.Suppose IR be the set of all real numbers and $f: IR \to IR$ is a function such that the following equations hold for all $x, y \in IR$:

(i) $f(x + y) = f(x) + f(y)$

(ii) $f(xy) = f(x)f(y)$

Show that $\forall x \in IR$ either $f(x) = 0, or, f(x) = x$. **[20M]**

UPSC – MATHEMATICS optional – 2019 Questions

1. Evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)} dx, \quad a > 0, a \neq 1.$ **[10M]**

2. Discuss the uniform convergence of $f_n(x) = \frac{nx}{1+n^2x^2}, \quad \forall\, x \in R(-\infty, \infty)$

$n = 1, 2, 3 \ldots.$ **[15M]**

3. Find the maximum value of $f(x, y, z) = x^2y^2z^2$ subject to the subsidiary condition $x^2 + y^2 + z^2 = c^2,\ (x, y, z > 0).$ **[15M]**

4. Discuss the convergence of $\int_1^2 \frac{\sqrt{x}}{l_n x} dx.$ **[15M]**