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Mathematics-Optional

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ABSTRACT ALGEBRA 2013-2019

UPSC – MATHEMATICS optional – 2013 Questions

1. Show that the set of matrices $S = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} | a, b \in R \right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$? Consider the map $f: C \to S$ defined by $f(a + ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that f is an isomorphism. (Here R is the set of real numbers and C is the set of complex numbers. **[10M]**

2. Give an example of an infinite group in which every element has finite order. **[10M]**

3. What are the orders of the following permutations in S_{10}?

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 7 & 3 & 10 & 5 & 4 & 2 & 6 & 9 \end{pmatrix}$ and $(1\,2\,3\,4\,5)(6\,7)$. **[10M]**

4. What is the maximal possible order of an element in S_{10}? Why? Give an example of such an element. How many elements will there be in S_{10} of that order? **[13M]**

5.Let $\text{J} = \{a + bi | a, b \,\epsilon Z\}$ be the ring of Gaussian integers (subring of C). Which of the following is J : Euclidean domain, principal ideal domain, unique factorization domain ? Justify your answer. **[15M]**

6. Let R^C = ring of all real valued continuous functions on [0, 1], under the operations.

$(f + g)x = f(x) + g(x)$

$(fg)x = f(x)g(x).$

$Let\;\; M = \left\{ f \in R^C \middle| f\left(\frac{1}{2}\right) = 0 \right\}.$

Is M a maximal ideal of R? Justify your answer. **[15M]**

UPSC – MATHEMATICS optional – 2014 Questions

1. Let G be the set of all real 2×2 matrices $\begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$, where $xz \neq 0$. Show that G is a group under matrix multiplication. Let N denote the subset $\left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in R \right\}$. Is N a normal subgroup of G? Justify your answer. **[10M]**

2. Show that Z_7 is a field. Then find $(|5| + |6|)^{-1}$ and $(-|4|)^{-1}$ in Z_7. **[15M]**

3. Show that the set $\{a + b\omega : \omega^3 = 1\}$, where a and b are real numbers, is a field with respect to usual addition and multiplication. **[15M]**

4. Prove that the set $Q(\sqrt{5}) = \{a + b\sqrt{5}: a, b \in Q\}$ is a commutative ring with identity.

[15M]

UPSC – MATHEMATICS optional – 2015 Questions

1. (i) How many generators are there of the cyclic group G of order 8? Explain.

(ii) Taking a group {e, a, b, c} of order 4, where e is the identity , construct composition tables showing that one is cyclic while the other is not. **[10M]**

2. Give an example of a ring having identity but a subring of this having a different identity.

[10M]

3. If R is a ring with unit element 1 *and* $\emptyset$ is homomorphism of R onto R' ,*prove that* $\emptyset(1)$ is the unit element of R'. **[15M]**

4. Do the following sets form integral domains with respect to ordinary addition and multiplication? If so, state if they are fields :

(i) The set of numbers of the form $b\sqrt{2}$ with b rational

(ii) The set of even integers

(iii) The set of positive integers **[15M]**

UPSC – MATHEMATICS optional – 2016 Questions

1. Let K be a field and K[X] be the ring of polynomials over K in a single variable X. For a polynomial $f \in K[X]$, let (f) denote the ideal in $K[X]$ generated by f. Show that (f) is a maximal ideal in K[X] if and only if f is an irreducible polynomial over K. **[10M]**

2. Let p be a prime number and Z_p denote the additive group of integers modulo p. Show that every non-zero element of Z_P generates Z_P. **[15M]**

3. Let K be an extension of a field F. Prove that the elements of K, which are algebraic over F, form a subfield of K. Further , if $F \subset K \subset L$ are fields, L is algebraic over K and K is algebraic over F, then prove that L, is algebraic over F. **[20M]**

4. Show that every algebraically closed field is infinite. **[15M]**

UPSC – MATHEMATICS optional – 2017 Questions

1. Let F be a field and F[X] denote the ring of polynomials over F in a single variable X. For f(X),g(X) $\in F[X]$ with g(X) $\neq 0$, show that there exist q(X),r(X) $\in F[X]$ such that degree (r(X))< degree (g(X)) and $f(X) = q(X).g(X) + r(X)$. **[20M]**

2. Let G be a group of order n. Show that G is isomorphic to a subgroup of the permutation group S_n. **[10M]**

3. Show that the groups $Z_5 \times Z_7$ *and* Z_{35} are isomorphic. **[15M]**

4. Let R be an Integral domain with unit element. Show that any unit in R[x] is a unit in R.

[10M]

UPSC – MATHEMATICS optional – 2018 Questions

1. Show that the quotient group of (lR, +) modulo Z is isomorphic to the multiplicative group of complex numbers on the unit circle in the complex plane. Here lR is the set of real numbers and Z is the set of integers. **[15M]**

2. Find all the proper subgroups of the multiplicative group of the field ($Z_{13}, +_{13}, \times_{13}$), where $+_{13}$ and $\times_{13}$ represent addition modulo 13 and multiplication modulo 13 respectively.

[15M]

UPSC – MATHEMATICS optional – 2019 Questions

1. Let G be a finite group, H and K subgroups of G such that $K \subset H$. Show that $(G:K) = (G:H)(H:K)$. **[10M]**

2. If G and H are finite groups whose orders are relatively prime, then prove that there is only one homomorphism from G to H, the trivial one. **[10M]**

3. Write down all quotient groups of the group Z_{12}. **[10M]**

4. Let a be an irreducible element of the Euclidean ring R, then prove that $R/(a)$ is a field.
[10M]