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Mathematics-Optional

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Vector Analysis 2013 - 2021

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UPSC – MATHEMATICS optional – 2013 Questions

- **1.** Show that the curve $\vec{x} = t\hat{\imath} + \left(\frac{1+t}{t}\right)\hat{\jmath} + \left(\frac{1-t^2}{t}\right)\hat{k}$ lies in a plane. [10M]
- 2. Calculate $\nabla^2(r^n)$ and find its expression in terms of *r* and *n*, *r* being the distance of any point (x, y, z) from the origin, *n* being a constant and ∇^2 being the Laplace operator.
- **3.** A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} t^3\hat{k}$. Determine the angle between the tangents to this curve at the points t = +1 and t = -1. [10M]
- 4. By using Divergence theorem of Gauss, evaluate the surface integral $\iint (a^2x^2 + b^2y^2 + c^2z^2)^{-\frac{1}{2}} ds$, where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. [15M]
- 5. Use Stokes' theorem to evaluate the line integral $\int_C (-y^3 dx + x^3 dy z^3 dz)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane x + y + z = 1. [15M]

UPSC – MATHEMATICS optional – 2014 Questions

- **1.** Find the curvature vector at any point of the curve $\bar{r}(t) = t \cos t \,\hat{i} + t \sin t \,\hat{j}, 0 \le t \le 2$ Give its magnitude also. [10M]
- 2. Evaluate by Stokes' theorem $\int_{\Gamma} (y \, dx + z \, dy + x \, dz)$ where Γ is the curve given by $x^2 + y^2 + z^2 2ax 2ay = 0$, x + y + 2a, starting from (2a, 0, 0) and then going below the z-plane. [20M]

UPSC – MATHEMATICS optional – 2015 Questions

- 1. A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{\imath} + (y^2 + x^2y)\hat{\jmath}$ Verify that the field \vec{F} is irrotational or not. Find the scalar potential. [12M]
- 2. Evaluate $\int_C e^{-x} (\sin y \, dx + \cos y \, dy)$, where C is the rectangle with vertices $(0,0), (\pi,0), (\pi,\frac{\pi}{2}), (0,\frac{\pi}{2}).$ [12M]

UPSC – MATHEMATICS optional – 2016 Questions

- 1. Prove that the vectors $\vec{a} = 3\hat{i} + \hat{j} 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} 2\hat{j} 6\hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle. [10M]
- 2. Find f(r) such that $\nabla f = \frac{\vec{r}}{r^5}$ and f(1) = 0. [10M]
- **3.** Prove that $\oint_C f \, d\vec{r} = \iint_S d\vec{S} \times \nabla f$ [10M]

[10M]

4. For the cardiod $r = a(1 + \cos \theta)$, show that the square of the radius of curvature at any point (r, θ) is proportional to *r*. Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$. [15M]

UPSC – MATHEMATICS optional – 2017 Questions

- 1. For what values of the constants *a*, *b* and *c* the vector $\overline{V} = (x + y + az)\hat{\iota} + (bx + 2y - z)\hat{j} + (-x + cy + 2z)\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values. [10M]
- 2. The position vector of a moving point at time t is $\bar{r} = \sin t \,\hat{i} + \cos 2t \,\hat{j} + (t^2 + 2t)\hat{k}$. Find the components of acceleration \bar{a} in the directions parallel to the velocity vector \bar{v} and perpendicular to the plane of \bar{r} and \bar{v} at time t = 0. [10M]
- 3. Find the curvature vector and its magnitude at any point $\bar{r} = (\theta)$ of the curve $\bar{r} = (a \cos \theta, a \sin \theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2 + y^2 z^2 = a^2$ [16M]
- 4. Evaluate the integral : $\iint_S \overline{F} \cdot \hat{n} ds$ where $\overline{F} = 3xy^2\hat{i} + (yx^2 y^3)\hat{j} + 3zx^2\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \le 4, -3 \le x \le 3$, using divergence theorem. [09M]
- 5. Using Green's theorem, evaluate the $\int_C F(\bar{r}) \cdot d\bar{r}$ counterclockwise where $F(\bar{r}) = (x^2 + y^2)\hat{i} + (x^2 y^2)\hat{j}$ and $d\bar{x} = dx\hat{y} + dy\hat{j}$ and the curve C is the boundary of the region $R = \{(x, y) | 1 \le y \le 2 x^2\}$. [08M]

UPSC – MATHEMATICS optional – 2018 Questions

- 1. If S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, then evaluate $\iint_S[(x+z)dydz + (y+z)dzdx + (x+y)dxdy]$ using Gauss' divergence theorem. [12M]
- 2. Find the curvature and torsion of the curve $\vec{r} = a(u \sin u)\vec{i} + a(1 \cos u)\vec{j} + bu\vec{k}$ [12M]
- 3. Let $\vec{v} = v_1 \vec{\iota} + v_2 \vec{j} + v_3 \vec{k}$. Show that $curl(curl \vec{v}) = grad(div \vec{v}) \nabla^2 \vec{v}$. [12M]
- 4. Evaluate the line integral $\int_C -y^3 dx + x^3 dy + z^2 dz$ using Stoke's theorem. Here C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane x + y + z = 1. The orientation on C corresponding to counterclockwise motion in the *xy*-plane. [13M]
- 5. Let $\vec{F} = xy^2\vec{\imath} + (y+x)\vec{j}$. Integrate $(\nabla \times \vec{F}) \cdot \vec{k}$ over the region in the first quadrant bounded by the curves $y = x^2$ and y = x using Green's theorem. [13M]

UPSC – MATHEMATICS optional – 2019 Questions

- 1. Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point (1, 1, 1). [10M]
- **2.** Find the circulation of \vec{F} round the curve *C*, where $\vec{F} = (2x + y^2)\hat{\imath} + (3y 4x)\hat{\jmath}$ and *C* is the curve $y = x^2$ from (0, 0) to (1, 1) to (0, 0). [15M]
- 3. Find the radius of curvature and radius of torsion of the helix $x = a \cos u$, $y = a \sin u$, $z = au \tan \alpha$. [15M]
- 4. State Gauss divergence theorem. Verify this theorem for $\vec{F} = 4x\hat{\imath} 2y^2\hat{\jmath} + z^2\hat{k}$, taken over the region bounded by $x^2 + y^2 = 4$, z = 0 and z = 3. [15M]
- 5. Evaluate by Stokes' theorem $\oint_C e^x dx + 2y dy dz$, where C is the curve $x^2 + y^2 = 4, z = 2.$ [05M]

UPSC – MATHEMATICS optional – 2020 Questions

- 1. For what value of *a*, *b*, *c* is the vector field $\overline{V} = (-4x - 3y + az)\hat{\imath} + (bx + 3y + 5z)\hat{\jmath} + (4x + cy + 3z)\hat{k}$ irrotational? Hence, express \overline{V} as the gradient of a scalar function ϕ . Determine ϕ . [10M]
- 2. For the vector function \overline{A} , where $\overline{A} = (3x^2 + 6y)\hat{i} 14yz\hat{j} + 20xz^2\hat{k}$, calculate $\int_C \overline{A} \cdot \overline{dr}$ from (0, 0, 0) to (1, 1, 1) along the following paths:
 - (i) $x = t, y = t^2, z = t^3$
 - (ii) Straight lines joining (0, 0, 0) to (1, 0, 0), then to (1, 1, 0) and then to (1, 1, 1)
 - (iii) Straight line joining (0,0,0) to (1,1,1) is the result same in all the cases? Explain the reason. [15M]
- 3. Verify the Stokes' theorem for the vector field $\overline{F} = xy \hat{i} + yz \hat{j} + xz \hat{k}$ on the surface S which is the part of the cylinder $z = 1 x^2$ for $0 \le x \le 1, -2 \le y \le 2$; S is oriented upwards. [20M]
- 4. Evaluate the surface integral $\iint_{S} \nabla \times \overline{F} \cdot \hat{n} \, ds$ for $\overline{F} = y\hat{i} + (x 2xz)\hat{j} xy\hat{k}$ and S is the surface of the sphere $x^{2} + y^{2} + z^{2} = a^{2}$ above the xy-plane. [15M]

UPSC – MATHEMATICS optional – 2021 Questions

- 1. Show that $\nabla^2 \left[\nabla \cdot \left(\frac{\vec{r}}{r} \right) \right] = \frac{2}{r^4}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. [10M]
- 2. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where *C* is an arbitrary closed curve in the *xy*-plane and $\vec{F} = \frac{-y\hat{\iota} + x\hat{j}}{x^2 + y^2}$

- 3. Verify Gauss divergence theorem for $\vec{F} = 2x^2y\hat{\imath} y^2\hat{\jmath} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and x = 2. [20M]
- 4. Using Stoke's theorem, evaluate $\iint_{S} (\nabla \times \vec{F}) \cdot \hat{n} dS$, where $\vec{F} = (x^{2} + y - 4)\hat{i} + 3xy\hat{j} + (2xy + z^{2})\hat{k}$ and S is the surface of the paraboloid $z = 4 - (x^{2} + y^{2})$ above the xy-plane. Here, \hat{n} is the unit outward normal vector on S.

[15M]