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Mathematics-Optional

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VECTOR ANALYSIS 2013-2019

UPSC – MATHEMATICS optional – 2013 Questions

1. Show that the curve $\vec{x} = t\hat{\imath} + \left(\frac{1+t}{t}\right)\hat{\jmath} + \left(\frac{1-t^2}{t}\right)\hat{k}$ lies in a plane. **[10M]**

2. Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x, y, z) from the origin, n being a constant and ∇^2 being the Laplace operator. **[10M]**

3. A curve in space is defined by the vector equation $\vec{r} = t^2\hat{\imath} + 2t\hat{\jmath} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. **[10M]**

4. By using Divergence theorem of Gauss, evaluate the surface integral $\iint (a^2x^2 + b^2y^2 + c^2z^2)^{-\frac{1}{2}} ds$, where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1, a, b$ and c being all positive constants. **[15M]**

5. Use Stokes' theorem to evaluate the line integral $\int_C (-y^3dx + x^3dy - z^3dz)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. **[15M]**

UPSC – MATHEMATICS optional – 2014 Questions

1. Find the curvature vector at any point of the curve $\bar{r}(t) = t\cos t\,\hat{\imath} + t\sin t\,\hat{\jmath}, 0 \le t \le 2$ Give its magnitude also. **[10M]**

2. Evaluate by Stokes' theorem $\int_\Gamma (y\,dx + z\,dy + x\,dz)$ where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y + 2a$, starting from $(2a, 0, 0)$ and then going below the z-plane. **[20M]**

UPSC – MATHEMATICS optional – 2015 Questions

1. A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{\imath} + (y^2 + x^2y)\hat{\jmath}$ Verify that the field $\vec{F}$ is irrotational or not. Find the scalar potential. **[12M]**

2. Evaluate $\int_C e^{-x}(\sin y\,dx + \cos y\,dy)$, where C is the rectangle with vertices $(0, 0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$. **[12M]**

UPSC – MATHEMATICS optional – 2016 Questions

1. Prove that the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k},\ \vec{b} = -\hat{\imath} + 3\hat{\jmath} + 4\hat{k}, \vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle. **[10M]**

2. Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1) = 0$. **[10M]**

3. Prove that $\oint_C f\,d\vec{r} = \iint_S d\vec{S} \times \nabla f$ **[10M]**

4. For the cardiod $r = a(1 + \cos\theta)$, show that the square of the radius of curvature at any point (r, θ) is proportional to r. Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$. **[15M]**

UPSC – MATHEMATICS optional – 2017 Questions

1. For what values of the constants a, b and c the vector $\bar{V} = (x + y + az)\hat{\imath} + (bx + 2y - z)\hat{\jmath} + (-x + cy + 2z)\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values. **[10M]**

2. The position vector of a moving point at time t is $\bar{r} = \sin t\,\hat{\imath} + \cos 2t\,\hat{\jmath} + (t^2 + 2t)\hat{k}$. Find the components of acceleration $\bar{a}$ in the directions parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time $t = 0$. **[10M]**

3. Find the curvature vector and its magnitude at any point $\bar{r} = (\theta)$ of the curve $\bar{r} = (a\cos\theta, a\sin\theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2 + y^2 - z^2 = a^2$ **[16M]**

4. Evaluate the integral : $\iint_S \bar{F} \cdot \hat{n} ds$ where $\bar{F} = 3xy^2\hat{\imath} + (yx^2 - y^3)\hat{\jmath} + 3zx^2\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \leq 4, -3 \leq x \leq 3$, using divergence theorem. **[09M]**

5. Using Green's theorem, evaluate the $\int_C F(\bar{r}) \cdot d\bar{r}$ counterclockwise where $F(\bar{r}) = (x^2 + y^2)\hat{\imath} + (x^2 - y^2)\hat{\jmath}$ and $d\bar{x} = dx\hat{y} + dy\hat{\jmath}$ and the curve C is the boundary of the region $R = \{(x, y) | 1 \leq y \leq 2 - x^2\}$. **[08M]**

UPSC – MATHEMATICS optional – 2018 Questions

1. If S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, then evaluate $\iint_S [(x + z)dydz + (y + z)dzdx + (x + y)dxdy]$ using Gauss' divergence theorem. **[12M]**

2. Find the curvature and torsion of the curve $\vec{r} = a(u - \sin u)\vec{\imath} + a(1 - \cos u)\vec{\jmath} + bu\,\vec{k}$ **[12M]**

3. Let $\vec{v} = v_1\vec{\imath} + v_2\vec{\jmath} + v_3\vec{k}$. Show that $curl\ (curl\ \vec{v}) = grad(div\ \vec{v}) - \nabla^2\vec{v}$. **[12M]**

4. Evaluate the line integral $\int_C -y^3dx + x^3dy + z^2dz$ using Stoke's theorem. Here C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. The orientation on C corresponding to counterclockwise motion in the xy-plane. **[13M]**

5. Let $\vec{F} = xy^2\vec{\imath} + (y + x)\vec{\jmath}$. Integrate $(\nabla \times \vec{F}) \cdot \vec{k}$ over the region in the first quadrant bounded by the curves $y = x^2$ and $y = x$ using Green's theorem. **[13M]**

UPSC – MATHEMATICS optional – 2019 Questions

1. Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point $(1, 1, 1)$. **[10M]**

2. Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = (2x + y^2)\hat{\imath} + (3y - 4x)\hat{\jmath}$ and C is the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$ to $(0, 0)$. **[15M]**

3. Find the radius of curvature and radius of torsion of the helix $x = a\cos u$, $y = a\sin u$, $z = au\tan\alpha$. **[15M]**

4. State Gauss divergence theorem. Verify this theorem for $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$, taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **[15M]**

5. Evaluate by Stokes' theorem $\oint_C e^x dx + 2y dy - dz$, where C is the curve $x^2 + y^2 = 4, z = 2$. **[05M]**