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## Mathematics-Optional By Venkanna Sir and Satya Sir

Vector Analysis 2013-2021

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## UPSC - MATHEMATICS optional - 2013 Questions

1. Show that the curve $\vec{x}=t \hat{\imath}+\left(\frac{1+t}{t}\right) \hat{\jmath}+\left(\frac{1-t^{2}}{t}\right) \hat{k}$ lies in a plane.
[10M]
2. Calculate $\nabla^{2}\left(r^{n}\right)$ and find its expression in terms of $r$ and $n, r$ being the distance of any point $(x, y, z)$ from the origin, $n$ being a constant and $\nabla^{2}$ being the Laplace operator.
[10M]
3. A curve in space is defined by the vector equation $\vec{r}=t^{2} \hat{\imath}+2 t \hat{\jmath}-t^{3} \hat{k}$. Determine the angle between the tangents to this curve at the points $t=+1$ and $t=-1$.
[10M]
4. By using Divergence theorem of Gauss, evaluate the surface integral $\iint\left(a^{2} x^{2}+b^{2} y^{2}+c^{2} z^{2}\right)^{-\frac{1}{2}} d s$, where S is the surface of the ellipsoid $a x^{2}+b y^{2}+c z^{2}=1, a, b$ and $c$ being all positive constants.
5. Use Stokes' theorem to evaluate the line integral $\int_{C}\left(-y^{3} d x+x^{3} d y-z^{3} d z\right)$, where $C$ is the intersection of the cylinder $x^{2}+y^{2}=1$ and the plane $x+y+z=1$.
[15M]

## UPSC - MATHEMATICS optional - 2014 Questions

1. Find the curvature vector at any point of the curve $\bar{r}(t)=t \cos t \hat{\imath}+t \sin t \hat{\jmath}, 0 \leq t \leq 2$ Give its magnitude also.
[10M]
2. Evaluate by Stokes' theorem $\int_{\Gamma}(y d x+z d y+x d z)$ where $\Gamma$ is the curve given by $x^{2}+y^{2}+z^{2}-2 a x-2 a y=0, x+y+2 a$, starting from $(2 a, 0,0)$ and then going below the z-plane.

## UPSC - MATHEMATICS optional - 2015 Questions

1. A vector field is given by $\vec{F}=\left(x^{2}+x y^{2}\right) \hat{\imath}+\left(y^{2}+x^{2} y\right) \hat{\jmath}$ Verify that the field $\vec{F}$ is irrotational or not. Find the scalar potential.
2. Evaluate $\int_{C} e^{-x}(\sin y d x+\cos y d y)$, where C is the rectangle with vertices

$$
\begin{equation*}
(0,0),(\pi, 0),\left(\pi, \frac{\pi}{2}\right),\left(0, \frac{\pi}{2}\right) \tag{12M}
\end{equation*}
$$

## UPSC - MATHEMATICS optional - 2016 Questions

1. Prove that the vectors $\vec{a}=3 \hat{\imath}+\hat{\jmath}-2 \hat{k}, \vec{b}=-\hat{\imath}+3 \hat{\jmath}+4 \hat{k}, \vec{c}=4 \hat{\imath}-2 \hat{\jmath}-6 \hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle.
[10M]
2. Find $f(r)$ such that $\nabla f=\frac{\vec{r}}{r^{5}}$ and $f(1)=0$.
3. Prove that $\oint_{C} f d \vec{r}=\iint_{S} d \vec{S} \times \nabla f$
4. For the cardiod $r=a(1+\cos \theta)$, show that the square of the radius of curvature at any point $(r, \theta)$ is proportional to $r$. Also find the radius of curvature if $\theta=0, \frac{\pi}{4}, \frac{\pi}{2}$.

## UPSC - MATHEMATICS optional - 2017 Questions

1. For what values of the constants $a, b$ and $c$ the vector
$\bar{V}=(x+y+a z) \hat{\imath}+(b x+2 y-z) \hat{\jmath}+(-x+c y+2 z) \hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values.
2. The position vector of a moving point at time $t$ is $\bar{r}=\sin t \hat{\imath}+\cos 2 t \hat{\jmath}+\left(t^{2}+2 t\right) \hat{k}$. Find the components of acceleration $\bar{a}$ in the directions parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time $t=0$.
[10M]
3. Find the curvature vector and its magnitude at any point $\bar{r}=(\theta)$ of the curve $\bar{r}=(a \cos \theta, a \sin \theta, a \theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^{2}+y^{2}-z^{2}=a^{2}$
[16M]
4. Evaluate the integral : $\iint_{S} \bar{F} \cdot \hat{n} d s$ where $\bar{F}=3 x y^{2} \hat{\imath}+\left(y x^{2}-y^{3}\right) \hat{\jmath}+3 z x^{2} \hat{k}$ and S is a surface of the cylinder $y^{2}+z^{2} \leq 4,-3 \leq x \leq 3$, using divergence theorem.
[09M]
5. Using Green's theorem, evaluate the $\int_{C} F(\bar{r}) \cdot d \bar{r}$ counterclockwise where $F(\bar{r})=\left(x^{2}+y^{2}\right) \hat{\imath}+\left(x^{2}-y^{2}\right) \hat{\jmath}$ and $d \bar{x}=d x \hat{y}+d y \hat{\jmath}$ and the curve C is the boundary of the region $R=\left\{(x, y) \mid 1 \leq y \leq 2-x^{2}\right\}$.
[08M]

## UPSC - MATHEMATICS optional - 2018 Questions

1. If $S$ is the surface of the sphere $x^{2}+y^{2}+z^{2}=a^{2}$, then evaluate $\iint_{S}[(x+z) d y d z+$ $(y+z) d z d x+(x+y) d x d y]$ using Gauss' divergence theorem.
2. Find the curvature and torsion of the curve $\vec{r}=a(u-\sin u) \vec{\imath}+a(1-\cos u) \vec{\jmath}+b u \vec{k}$
3. Let $\vec{v}=v_{1} \vec{\imath}+v_{2} \vec{\jmath}+v_{3} \vec{k}$. Show that $\operatorname{curl}(\operatorname{curl} \vec{v})=\operatorname{grad}(\operatorname{div} \vec{v})-\nabla^{2} \vec{v}$.
4. Evaluate the line integral $\int_{C}-y^{3} d x+x^{3} d y+z^{2} d z$ using Stoke's theorem. Here C is the intersection of the cylinder $x^{2}+y^{2}=1$ and the plane $x+y+z=1$. The orientation on C corresponding to counterclockwise motion in the $x y$-plane.
5. Let $\vec{F}=x y^{2} \vec{\imath}+(y+x) \vec{\jmath}$. Integrate $(\nabla \times \vec{F}) \cdot \vec{k}$ over the region in the first quadrant bounded by the curves $y=x^{2}$ and $y=x$ using Green's theorem.

## UPSC - MATHEMATICS optional - 2019 Questions

1. Find the directional derivative of the function $x y^{2}+y z^{2}+z x^{2}$ along the tangent to the curve $x=t, y=t^{2}, z=t^{3}$ at the point $(1,1,1)$.
[10M]
2. Find the circulation of $\vec{F}$ round the curve $C$, where $\vec{F}=\left(2 x+y^{2}\right) \hat{\imath}+(3 y-4 x) \hat{\jmath}$ and $C$ is the curve $y=x^{2}$ from $(0,0)$ to $(1,1)$ to $(0,0)$.
[15M]
3. Find the radius of curvature and radius of torsion of the helix $x=a \cos u, y=a \sin u$, $z=a u \tan \alpha$.
[15M]
4. State Gauss divergence theorem. Verify this theorem for $\vec{F}=4 x \hat{\imath}-2 y^{2} \hat{\jmath}+z^{2} \hat{k}$, taken over the region bounded by $x^{2}+y^{2}=4, z=0$ and $z=3$.
5. Evaluate by Stokes' theorem $\oint_{C} e^{x} d x+2 y d y-d z$, where $C$ is the curve $x^{2}+y^{2}=4, z=2$.
[05M]

## UPSC - MATHEMATICS optional - 2020 Questions

1. For what value of $a, b, c$ is the vector field
$\bar{V}=(-4 x-3 y+a z) \hat{\imath}+(b x+3 y+5 z) \hat{\jmath}+(4 x+c y+3 z) \hat{k}$ irrotational? Hence, express $\bar{V}$ as the gradient of a scalar function $\phi$. Determine $\phi$.
[10M]
2. For the vector function $\bar{A}$, where $\bar{A}=\left(3 x^{2}+6 y\right) \hat{\imath}-14 y z \hat{\jmath}+20 x z^{2} \widehat{k}$, calculate $\int_{C} \bar{A} \cdot \overline{d r}$ from $(0,0,0)$ to $(1,1,1)$ along the following paths:
(i) $x=t, y=t^{2}, z=t^{3}$
(ii) Straight lines joining $(0,0,0)$ to $(1,0,0)$, then to $(1,1,0)$ and then to $(1,1,1)$
(iii) Straight line joining $(0,0,0)$ to $(1,1,1)$ is the result same in all the cases? Explain the reason.
[15M]
3. Verify the Stokes' theorem for the vector field $\bar{F}=x y \hat{\imath}+y z \hat{\jmath}+x z \hat{k}$ on the surface $S$ which is the part of the cylinder $z=1-x^{2}$ for $0 \leq x \leq 1,-2 \leq y \leq 2 ; S$ is oriented upwards
4. Evaluate the surface integral $\iint_{S} \nabla \times \bar{F} \cdot \hat{n} d s$ for $\bar{F}=y \hat{\imath}+(x-2 x z) \hat{\jmath}-x y \hat{k}$ and $S$ is the surface of the sphere $x^{2}+y^{2}+z^{2}=a^{2}$ above the $x y$-plane.
[15M]

## UPSC - MATHEMATICS optional - 2021 Questions

1. Show that $\nabla^{2}\left[\nabla \cdot\left(\frac{\vec{r}}{r}\right)\right]=\frac{2}{r^{4}}$, where $\vec{r}=x \hat{\imath}+y \hat{\jmath}+z \hat{k}$.
2. Evaluate $\int_{C} \vec{F} \cdot d \vec{r}$, where $C$ is an arbitrary closed curve in the $x y$-plane and $\vec{F}=\frac{-y \hat{\imath}+x \hat{\jmath}}{x^{2}+y^{2}}$
3. Verify Gauss divergence theorem for $\vec{F}=2 x^{2} y \hat{\imath}-y^{2} \hat{\jmath}+4 x z^{2} \hat{k}$ taken over the region in the first octant bounded by $y^{2}+z^{2}=9$ and $x=2$.
[20M]
4. Using Stoke's theorem, evaluate $\iint_{S}(\nabla \times \vec{F}) \cdot \hat{n} d S$,
where $\vec{F}=\left(x^{2}+y-4\right) \hat{\imath}+3 x y \hat{\jmath}+\left(2 x y+z^{2}\right) \hat{k}$ and $S$ is the surface of the paraboloid $z=4-\left(x^{2}+y^{2}\right)$ above the $x y$-plane. Here, $\hat{n}$ is the unit outward normal vector on $S$.
