

# **INDIANCIVILS.COM**

## **Online IAS Academy**

# **Mathematics-Optional**

**By Venkanna Sir and Satya Sir**

**Dynamics & Statics 2013 - 2021**

**INDIANCIVILS.COM +91-9000018804**

## UPSC – MATHEMATICS optional – 2013 Questions

1. A body is performing S.H.M. in a straight line OPQ. Its velocity is zero at points  $P$  and  $Q$  whose distances from  $O$  are  $x$  and  $y$  respectively and its velocity is  $v$  at the mid-point between  $P$  and  $Q$ . Find the time of one complete oscillation. [10M]
2. The base of an inclined plane is 4 metres in length and the height is 3 metres. A force of 8 kg acting parallel to the plane will just prevent a weight of 20 kg from sliding down. Find the coefficient of friction between the plane and the weight. [10M]
3. A particle of mass  $2.5 \text{ kg}$  hangs at the end of a string  $0.9 \text{ m}$  long, the other end of which is attached to a fixed point. The particle is projected horizontally with a velocity  $8 \text{ m/sec}$ . Find the velocity of the particle and tension in the string when the string is (i) horizontal (ii) vertically upward. [20M]
4. A uniform ladder rests at an angle of  $45^\circ$  with the horizontal with its upper extremity against a rough vertical wall and its lower extremity on the ground. If  $\mu$  and  $\mu'$  are the coefficients of limiting friction between the ladder and the ground and wall respectively, then find the minimum horizontal force required to move the lower end of the ladder towards the wall. [15M]
5. Six equal rods  $AB, BC, CD, DE, EF$  and  $FA$  are each of weight  $W$  and are freely jointed at their extremities so as to form a hexagon; the rod  $AB$  is fixed in a horizontal position and the middle points of  $AB$  and  $DE$  are joined by a string. Find the tension in the string. [15M]
6. Two equal rods  $AB$  and  $BC$ , each of length  $l$ , smoothly jointed at  $B$ , are suspended from  $A$  and oscillate in a vertical plane through  $A$ . Show that the periods of normal oscillations are  $\frac{2\pi}{n}$  where  $n^2 = \left(3 + \frac{6}{\sqrt{7}}\right) \frac{g}{l}$ . [15M]

## UPSC – MATHEMATICS optional – 2014 Questions

1. A particle is performing a simple harmonic motion (S.H.M). of period  $T$  about a centre  $O$  with amplitude  $a$  and it passes through a point  $P$ , where  $OP = b$  in the direction  $OP$ . Prove that the time which elapses before it returns to  $P$  is  $\frac{T}{\pi} \cos^{-1} \left( \frac{b}{a} \right)$ . [10M]
2. Two equal uniform rods  $AB$  and  $AC$ , each of length  $l$ , are freely jointed at  $A$  and rest on a smooth fixed vertical circle of radius  $r$ . If  $2\theta$  is the angle between the rods, then find the relation between  $l, r$  and  $\theta$ , by using the principle of virtual work. [10M]

3. A particle of mass  $m$ , hanging vertically from a fixed point by a light inextensible cord of length  $l$ , is struck by a horizontal blow which imparts to it a velocity  $2\sqrt{gl}$ . Find the velocity and height of the particle from the level of its initial position when the cord becomes slack. [15M]
4. A regular pentagon ABCDE, formed of equal heavy uniform bars jointed together, is suspended from the joint A, and is maintained in form by a light rod joining the middle points of BC and DE. Find the stress in this rod. [20M]
5. A particle is acted on by a force parallel to the axis of  $y$  whose acceleration (always towards the axis of  $x$ ) is  $\mu y^2$  and when  $y = a$ , it is projected parallel to the axis of  $x$  with velocity  $\sqrt{\frac{2\mu}{a}}$ . Find the parametric equation of the path of the particle. Here  $\mu$  is a constant. [15M]

### UPSC – MATHEMATICS optional – 2015 Questions

1. A body moving under SHM has an amplitude 'a' and time period 'T'. If the velocity is trebled, when the distance from mean position is ' $\frac{2}{3}a$ ', the period being unaltered, find the new amplitude. [10M]
2. A rod of 8 kg is movable in a vertical plane about a hinge at one end, another end is fastened a weight equal to half of the rod, this end is fastened by a string of length  $l$  to a point at a height  $b$  above the hinge vertically. Obtain the tension in the string. [10M]
3. Two equal ladders of weight 4 kg each are placed so as to lean at A against each other with their ends resting on a rough floor, given the coefficient of friction is  $\mu$ . The ladders at A make an angle  $60^\circ$  with each other. Find what weight on the top would cause them to slip. [13M]
4. A mass starts from rest at a distance 'a' from the centre of force which attracts inversely as the distance. Find the time of arriving at the centre. [13M]
5. A particle is projected from the base of a hill whose slope is that of a right circular cone, whose axis is vertical. The projectile grazes the vertex and strikes the hill again at a point on the base. If the semivertical angle of the cone is  $30^\circ$ ,  $h$  is height, determine the initial velocity  $u$  of the projection and its angle of projection. [13M]
6. Find the length of an endless chain which will hang over a circular pulley of radius 'a' so as to be in contact with the two-thirds of the circumference of the pulley. [12M]
7. A particle moves in a plane under a force, towards a fixed centre, proportional to the distance. If the path of the particle has two apsidal distances  $a, b$  ( $a > b$ ), then find the equation of the path. [13M]

## UPSC – MATHEMATICS optional – 2016 Questions

1. A particle moves with a central acceleration which varies inversely as the cube of the distance. If it is projected from an apse at a distance  $a$  from the origin with a velocity which is  $\sqrt{2}$  times the velocity for a circle of radius  $a$ , then find the equation to the path. [10M]
2. A uniform rod AB of length  $2a$  movable about a hinge at A rests with other end against a smooth vertical wall. If  $\alpha$  is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is  $\frac{1}{2}W\sqrt{4 + \tan^2\alpha}$  where  $W$  is the weight of the rod. [15M]
3. Two weights P and Q are suspended from a fixed point O by strings OA, OB and are kept apart by a light rod AB. If the strings OA and OB make angles  $\alpha$  and  $\beta$  with the rod AB, show that the angle  $\theta$  which the rod makes with the vertical is given by

$$\tan \theta = \frac{P+Q}{P \cot \alpha - Q \cot \beta} \quad [15M]$$

4. A square ABCD, the length of whose sides is  $a$ , is fixed in a vertical plane with two of its sides horizontal. An endless string of length  $l (> 4a)$  passes over four pegs at the angles of the board and through a ring of weight  $W$  which is hanging vertically. Show that the tension of the string is  $\frac{W(l-3a)}{2\sqrt{l^2-6la+8a^2}}$  [20M]
5. A particle moves in a straight line. Its acceleration is directed towards a fixed point O in the line and is always equal to  $\mu \left(\frac{a^5}{x^2}\right)^{1/3}$  when it is at a distance  $x$  from O. If it starts from rest at a distance  $a$  from O, then find the time, the particle will arrive at O. [15M]

## UPSC – MATHEMATICS optional – 2017 Questions

1. A fixed wire is in the shape of the cardioid  $r = a(1 + \cos \theta)$ , the initial line being the downward vertical. A small ring of mass  $m$  can slide on the wire and is attached to the point  $r = 0$  of the cardioid by an elastic string of natural length  $a$  and modulus of elasticity  $4mg$ . The string is released from rest when the string is horizontal. Show by using the laws of conservation of energy that  $a\theta^2(1 + \cos \theta) - g \cos \theta(1 - \cos \theta) = 0$ ,  $g$  being the acceleration due to gravity. [10M]
2. A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle  $\phi$  with its curved surface touching the plane. Find the greatest admissible value of the inclination  $\phi$  with its curved surface touching the plane. Find the greatest admissible value of the inclination  $\phi$  for equilibrium. If  $\phi$  be less than this value, is the equilibrium stable? [17M]

3. A particle is free to move on a smooth vertical circular wire of radius  $\alpha$ . At time  $t = 0$  it is projected along the circle from its lowest point A with velocity just sufficient to carry it to the highest point B. Find the time T at which the reaction between the particle and the wire is zero. [17M]
4. A spherical shot of W gm weight and radius  $r$  cm, lies at the bottom of cylindrical bucket of radius  $R$  cm. The bucket is filled with water up to a depth of  $h$  cm ( $h > 2r$ ). Show that the minimum amount of work done in lifting the shot just clear of the water must be  $\left[ W \left[ h - \frac{4r^3}{3R^2} \right] + W' \left[ r - h + \frac{2r^3}{3R^2} \right] \right]$  cm gm.  $W'$  gm is the weight of water displaced by the shot. [16M]

### UPSC – MATHEMATICS optional – 2018 Questions

1. A particle projected from a given point on the ground just clears a wall of height  $h$  at a distance  $d$  from the point of projection. If the particle moves in a vertical plane and if the horizontal range is R, find the elevation of the projection. [10M]
2. A particle moving with simple harmonic motion in a straight line velocities  $v_1$  and  $v_2$  at distances  $x_1$  and  $x_2$  respectively from the centre of its path. Find the period of its motion. [12M]

### UPSC – MATHEMATICS optional – 2019 Questions

1. One end of a heavy uniform rod AB can slide along a rough horizontal rod AC, to which it is attached by a ring. B and C are joined by a string. When the rod is on the point of sliding, then  $AC^2 - AB^2$ . If  $\theta$  is the angle between AB and the horizontal line, then prove that the coefficient of friction is  $\frac{\cot \theta}{2 + \cot^2 \theta}$ . [10M]
2. The force of attraction of a particle by the earth is inversely proportional to the square of its distance from the earth's Centre. A particle, whose weight on the surface of the earth is W, falls to the surface of the earth from a height  $3h$  above it. Show that the magnitude of work done by the earth's attraction force is  $\frac{3}{4}hW$ , where  $h$  is the radius of the earth. [10M]
3. A body consists of a cone and underlying hemisphere. The base of the cone and the top of the hemisphere have same radius  $a$ . The whole body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is  $\sqrt{3}a$ . [15M]

4. A particle moving along the  $y$ -axis has an acceleration  $Fy$  towards the origin, where  $F$  is a positive and even function of  $y$ . The periodic time, when the particle vibrates between  $y = -a$  and  $y = a$ , is  $T$ . Show that

$$\frac{2\pi}{\sqrt{F_1}} < T < \frac{2\pi}{\sqrt{F_2}}$$

where  $F_1$  and  $F_2$  are the greatest and the least values of  $F$  within the range  $[-a, a]$ .

Further, show that when a simple pendulum of length  $l$  oscillates through  $30^\circ$  on either side of the vertical line,  $T$  lies between  $2\pi\sqrt{l/g}$  and  $2\pi\sqrt{l/g}\sqrt{\pi/3}$ . [20M]

5. Prove that the path of a planet, which is moving so that its acceleration is always directed to a fixed point (start) and is equal to  $\frac{\mu}{(\text{distance})^2}$ , is a conic section. Find the conditions under which the path becomes (i) ellipse, (ii) parabola and (iii) hyperbola. [15M]

### UPSC – MATHEMATICS optional – 2020 Questions

1. A uniform rod, in vertical position, can turn freely about one of its ends and is pulled aside from the vertical by a horizontal force acting at the other end of the rod and equal to half its weight. At what inclination to the vertical will the rod rest? [10M]
2. A light rigid rod ABC has three particles each of mass  $m$  attached to it at  $A, B$  and  $C$ . The rod is struck by a blow  $P$  at right angles to it at a point distant from  $A$  equal to  $BC$ . Prove that the kinetic energy set up is  $\frac{1}{2} \frac{P^2}{m} \frac{a^2 - ab + b^2}{a^2 + ab + b^2}$ , where  $AB = a$  and  $BC = b$ . [10M]
3. A beam  $AD$  rests on two supports  $B$  and  $C$ , where  $AB = BC = CD$ . It is found that the beam will tilt when a weight of  $p$  kg is hung from  $A$  or when a weight of  $q$  kg is hung from  $D$ . Find the weight of the beam. [15M]
4. A square framework formed of uniform heavy rods of equal weight  $W$  jointed together, is hung up by one corners and the shape of the square is preserved by a light rod along the horizontal diagonal. Find the thrust of the light rod. [10M]
5. A particle starts at a great distance with velocity  $V$ . Let  $p$  be the length of the perpendicular from the centre of a star on the tangent to the initial path of the particle. Show that the least distance of the particle from the centre of the star is  $\lambda$ , where  $V^2\lambda = \sqrt{\mu^2 + p^2V^4} - \mu$ . Here  $\mu$  is a constant. [10M]
6. A four-wheeled railway truck has a total mass  $M$ , the mass and radius of gyration of each pair of wheels and axle are  $m$  and  $k$  respectively, and the gyration of each wheel is  $r$ . Prove that if the truck is propelled along a level track by a force  $P$ , the acceleration is  $\frac{P}{M + \frac{2mk^2}{r^2}}$ , and find the horizontal force exerted on each axle by the truck. The axle friction and wind resistance are to be neglected. [15M]

## UPSC – MATHEMATICS optional – 2021 Questions

- Two rods  $LM$  and  $MN$  are joined rigidly at the point  $M$  such that  $(LM)^2 + (MN)^2 = (LN)^2$  and they are hanged freely in equilibrium from a fixed point  $L$ . Let  $\omega$  be the weight per unit length of both the rods which are uniform. Determine the angle, which the rod  $LM$  makes with the vertical direction, in terms of lengths of the rods. [10M]
- A heavy string, which is not of uniform density, is hung up from two points. Let  $T_1, T_2, T_3$  be the tensions at the intermediate points  $A, B, C$  of the catenary respectively where its inclinations to the horizontal are in arithmetic progression with common difference  $\beta$ . Let  $\omega_1$  and  $\omega_2$  be the weights of the parts  $AB$  and  $BC$  of the string respectively. Prove that
  - Harmonic mean of  $T_1, T_2$  and  $T_3 = \frac{3T_2}{1+2\cos\beta}$
  - $\frac{T_1}{T_3} = \frac{\omega_1}{\omega_2}$[20M]
- A heavy particle hangs by an inextensible string of length  $a$  from a fixed point and is then projected horizontally with a velocity  $\sqrt{2gh}$ . If  $\frac{5a}{2} > h > a$ , then prove that the circular motion ceases when the particle has reached the height  $\frac{1}{3}(a + 2h)$  from the point of projection. Also, prove that the greatest height ever reached by the article above the point of projection is  $\frac{(4a-h)(a+2h)^2}{27a^2}$ . [15M]
- Describe the motion and path of a particle of mass  $m$  which is projected in a vertical plane through a point of projection with velocity  $u$  in a direction making an angle  $\theta$  with the horizontal direction. Further, if particles are projected from that point in the same vertical plane with velocity  $4\sqrt{g}$ , then determine the locus of vertices of their paths. [15M]
- If a planet, which revolves around the Sun in a circular orbit, is suddenly stopped in its orbit, then find the time in which it would fall into the Sun. Also, find the ration of its falling time to the period of revolution of the planet. [10M]