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## Mathematics-Optional

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## Ordinary Differential Equations2013-2021

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## UPSC - MATHEMATICS optional - 2013 Questions

1. $y$ is a function of $x$, such that the differential coefficient $\frac{d y}{d x}$ is equal to $\cos (x+y)+\sin (x+y)$. Find out a relation between $x$ and $y$, which is free from any derivative/differential.
2. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^{2}=a \sin n \theta,(r, \theta)$ being the plane polar coordinates.
3. Solve the differential equation $\left(5 x^{3}+12 x^{2}+6 y^{2}\right) d x+6 x y d y=0$.
4. Using the method of variation of parameters, solve the differential equation $\frac{d^{2} y}{d x^{2}}+a^{2} y=\sec a x$.
5. Find the general solution of the equation $x^{2} \frac{d^{2} y}{d x^{2}}+x \frac{d y}{d x}+y=\ln x \sin (\ln x)$
6. By using Laplace transform method, solve the differential equation $\left(D^{2}+n^{2}\right) x=a \sin (n t+\alpha), D^{2}=\frac{d^{2}}{d t^{2}}$ subject to the initial conditions $x=0$ and $\frac{d x}{d t}=$ 0 , at $t=0$, in which $a, n$ and $\alpha$ are constants.

## UPSC - MATHEMATICS optional - 2014 Questions

1. Justify that a differential equation of the form:
$\left[y+x f\left(x^{2}+y^{2}\right)\right] d x+\left[y+y f\left(x^{2}+y^{2}\right)-x\right] d y=0$, where $f\left(x^{2}+y^{2}\right)$ is an arbitrary function of $\left(x^{2}+y^{2}\right)$, is not an exact differential equation and $\frac{1}{x^{2}+y^{2}}$ is an integrating factor for it. Hence solve this differential equation for $f\left(x^{2}+y^{2}\right)=\left(x^{2}+y^{2}\right)^{2}$.
2. Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency.
[10M]
3. Solve by the method of variation of parameters. $\frac{d y}{d x}-5 y=\sin x$
4. Solve the differential equation $x^{3} \frac{d^{3} y}{d x^{3}}+3 x^{2} \frac{d^{2} y}{d x^{2}}+x \frac{d y}{d x}+8 y=65 \cos \left(\log _{\mathrm{e}} \mathrm{x}\right) \quad$ [20M]
5. Solve the following differential equation: $x \frac{d^{2} y}{d x^{2}}-2(x+1) \frac{d y}{d x}+(x+2) y=(x-2) e^{2 x}$ When $e^{x}$ is a solution to its corresponding homogeneous differential equation.
[15M]
6. Find the sufficient condition for the differential equation $M(x, y) d x+N(x, y) d y=0$ to have an integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence find the integrating factor for the differential equation $\left(x^{2}+x y\right) d x+\left(y^{2}+x y\right) d y=0$, and solve it.
7. Solve the initial value problem $\frac{d^{2} y}{d t^{2}}+y=8 e^{-2 t} \sin t, y(0)=0, y^{\prime}(0)=0$ by using Laplace-transform.
[20M]

## UPSC - MATHEMATICS optional - 2015 Questions

1. Solve the differential equation: $x \cos x \frac{d y}{d x}+y(x \sin x+\cos x)=1$.
2. Solve the differential equation:

$$
\begin{equation*}
\left(2 x y^{4} e^{y}+2 x y^{3}+y\right) d x+\left(x^{2} y^{4} e^{y}-x^{2} y^{2}-3 x\right) d y=0 \tag{10M}
\end{equation*}
$$

3. Find the angle between the surfaces $x^{2}+y^{2}+z^{2}-9=0$ and $z=x^{2}+y^{2}-3$ at $(2,-1,2)$.
4. Find the constant a so that $(x+y)^{a}$ is the integrating factor of $\left(4 x^{2}+2 x y+6 y\right) d x+\left(2 x^{2}+9 y+3 x\right) d y=0$ and hence solve the differential equation.
5. Find the value of $\lambda$ and $\mu$ so that the surfaces $\lambda x^{2}-\mu y z=(\lambda+2) x$ and $4 x^{2} y+z^{3}=4$ may intersect orthogonally at $(1,-1,2)$
[12M]
6. (i) Obtain Laplace inverse transform of $\left\{\ln \left(1+\frac{1}{s^{2}}\right)+\frac{s}{s^{2}+25} e^{-R s}\right\}$
(ii) Using Laplace transform, solve $y^{\prime \prime}+y=t, y(0)=1, y^{\prime}(0)=-2$.
7. Solve the differential equation $x=p y-p^{2}$ where $p=\frac{d y}{d x}$
8. Solve $x^{4} \frac{d^{4} y}{d x^{4}}+6 x^{3} \frac{d^{3} y}{d x^{3}}+4 x^{2} \frac{d^{2} y}{d x^{2}}-2 x \frac{d y}{d x}-4 y=x^{2}+2 \cos \left(\log _{e} x\right)$.

## UPSC - MATHEMATICS optional - 2016 Questions

1. Find a particular integral of $\frac{d^{2} y}{d x^{2}}+y=e^{x / 2} \sin \frac{x \sqrt{3}}{2}$
2. Show that the family of parabolas $y^{2}=4 c x+4 c^{2}$ is self-orthogonal
3. Solve $\left\{y(1-x \tan x)+x^{2} \cos x d x-x d y=0\right\}$
4. Using the method of variation of parameters, solve the differential equation

$$
\begin{equation*}
\left(D^{2}+2 D+1\right) y=e^{-x} \log (x),\left[D=\frac{d}{d x}\right] \tag{15M}
\end{equation*}
$$

5. Find the general solution of the equation $x^{2} \frac{d^{3} y}{d x^{3}}-4 x \frac{d^{2} y}{d x^{2}}+6 \frac{d y}{d x}=4$.
6. Using Laplace transformation, solve the following: $y^{\prime \prime}-2 y^{\prime}-8 y=0, y(0)=3$,

$$
y^{\prime}(0)=6
$$

[10M]

## UPSC - MATHEMATICS optional - 2017 Questions

1. Find the differential equation representing all the circles in the $x-y$ plane.
[10M]
2. Suppose that the streamlines of the fluid flow are given by a family of curves $x y=c$. Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines.
3. Solve the following simultaneous linear differential equations:
$(D+1) y=z+e^{x}$ and $(D+1) z+y+e^{x}$ where $y$ and $z$ are functions of independent variable $x$ and $D \equiv \frac{d}{d x}$.
[08M]
4. If the growth rate of the population of bacteria at any time $t$ is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks?
[08M]
5. Consider the differential equation $x y p^{2}-\left(x^{2}+y^{2}-1\right) p+x y=0$ where $p=\frac{d y}{d x}$. Substituting $u=x^{2}$ and $v=y^{2}$ reduce the equation to Clairaut's form in terms of $u, v$ and $p^{\prime}=\frac{d v}{d u}$. Hence, or otherwise solve the equation.
6. Solve the following initial value differential equations:

$$
\begin{equation*}
20 y^{\prime \prime}+4 y^{\prime}+y=0, y(0)=3.2 \text { and } y^{\prime}(0)=0 \tag{07M}
\end{equation*}
$$

7. Solve the differential equation: $x \frac{d^{2} y}{d x^{2}}-\frac{d y}{d x}-4 x^{3} y=8 x^{3} \sin \left(x^{2}\right)$
8. Solve the following differential equation using method of variation of parameters:

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}-\frac{d y}{d x}-2 y=44-76 x-48 x^{2} \tag{08M}
\end{equation*}
$$

9. Solve the following initial value problem using Laplace transform:

$$
\frac{d^{2} y}{d x^{2}}+9 y=r(x), y(0)=0, y^{\prime}(0)=4 \text { where } r(x)=\left\{\begin{array}{cl}
8 \sin x & \text { if } 0<x<\pi \\
0 & \text { if } x \geq \pi
\end{array}\right.
$$

## UPSC - MATHEMATICS optional - 2018 Questions

1. Find the angle between the tangent at a general point of the curve whose equations are $x=3 t, y=3 t^{2}, z=3 t^{3}$ and the line $y=z-x=0$,
2. Solve $y^{\prime \prime \prime}-6 y^{\prime \prime}+12 y^{\prime}-8 y=12 e^{2 x}+27 e^{-x}$
3. (i) Find the Laplace transform of $f(t)=\frac{1}{\sqrt{t}}$.
(ii) Find the inverse Laplace transform of $\frac{5 s^{2}+3 s-16}{(s-1)(s-2)(s+3)}$
[10M]
4. Solve $y^{\prime \prime}-y=x^{2} e^{2 x}$
5. Solve $\left(\frac{d y}{d x}\right)^{2} y+2 \frac{d y}{d x} x-y=0$
6. Solve $y^{\prime \prime}+16 y=32 \sec 2 x$
7. Solve $(1+x)^{2} y^{\prime \prime}+(1+x) y^{\prime}+y=4 \cos (\log (1+x))$
8. Solve the initial value problem

$$
\begin{aligned}
& y^{\prime \prime}-5 y^{\prime}+4 y=e^{2 t} \\
& y(0)=\frac{19}{12}, y^{\prime}(0)=\frac{8}{3}
\end{aligned}
$$

9. Find $\alpha$ and $\beta$ such that $x^{\alpha} y^{\beta}$ is an integrating factor of $\left(4 y^{2}+3 x y\right) d x-\left(3 x y+2 x^{2}\right) d y=0$ and solve the equation.
[12M]
10. Find $f(y)$ such that $\left(2 x e^{y}+3 y^{2}\right) d y+\left(3 x^{2}+f(y)\right) d x=0$ is exact and hence solve.
[12M]

## UPSC - MATHEMATICS optional - 2019 Questions

1. Solve the differential equation

$$
\begin{equation*}
\left(2 y \sin x+3 y^{4} \sin x \cos x d x\right)-\left(4 y^{3} \cos ^{2} x+\cos x\right) d y=0 \tag{10M}
\end{equation*}
$$

2. Determine the complete solution of the differential equation

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}-4 \frac{d y}{d x}+4 y=3 x^{2} e^{2 x} \sin 2 x \tag{10M}
\end{equation*}
$$

3. Solve the differential equation

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}+(3 \sin x-\cot x) \frac{d y}{d x}+2 y \sin ^{2} x=e^{-\cos x} \sin ^{2} x \tag{10M}
\end{equation*}
$$

4. Find the Laplace transforms of $t^{-1 / 2}$ and $t^{1 / 2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2^{\prime}}}$ where $n \in N$, is $\frac{\Gamma\left[n+1+\frac{1}{2}\right]}{s^{n+1+\frac{1}{2}}}$
5. Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^{2} y^{\prime \prime}-2 x y^{\prime}+2 y=x^{3} \sin x$ and then find the general solution of the given equation by the method of variation of parameters.
6. Obtain the singular solution of the differential equation $\left(\frac{d y}{d x}\right)^{2}\left(\frac{y}{x}\right)^{2} \cot ^{2} \alpha-2\left(\frac{d y}{d x}\right)\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^{2} \operatorname{cosec}^{2} \alpha=1$ Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solution.

## UPSC - MATHEMATICS optional - 2020 Questions

1. Solve the following differential equation:

$$
\begin{equation*}
x \cos \left(\frac{y}{x}\right)(y d x+x d y)=y \sin \left(\frac{y}{x}\right)(x d y-y d x) \tag{10M}
\end{equation*}
$$

2. Find the orthogonal trajectories of the family of circles passing through the points $(0,2)$ and $(0,-2)$.
3. Using the method of variation of parameters, solve the differential equation

$$
\begin{equation*}
y^{\prime \prime}+(1-\cot x) y^{\prime}-y \cot x=\sin ^{2} x, \text { if } y=e^{-x} \text { is one solution of } \mathrm{CF} . \tag{20M}
\end{equation*}
$$

4. Using Laplace transform, solve the initial value problem $t y^{\prime \prime}+2 t y^{\prime}+2 y=2$; $y(0)=1$ and $y^{\prime}(0)$ is arbitrary. Does this problem have a unique solution?
5. Solve the following differential equation:

$$
\begin{equation*}
(x+1)^{2} y^{\prime \prime}-4(x+1) y^{\prime}+6 y=6(x+1)^{2}+\sin \log (x+1) \tag{10M}
\end{equation*}
$$

6. Find the general and singular solutions of the differential equation

$$
\begin{equation*}
9 p^{2}(2-y)^{2}=4(3-y), \text { where } p=\frac{d y}{d x} . \tag{10M}
\end{equation*}
$$

## UPSC - MATHEMATICS optional - 2021 Questions

1. Solve the differential equation: $\frac{d^{2} y}{d x^{2}}+2 y=x^{2} e^{3 x}+e^{x} \cos 2 x$
2. Solve the initial value problem: $\frac{d^{2} y}{d x^{2}}+4 y=e^{-2 x} \sin 2 x ; y(0)=y^{\prime}(0)=0$ using Laplace transform method.
3. Solve the equation: $\frac{d^{2} y}{d x^{2}}+(\tan x-3 \cos x) \frac{d y}{d x}+2 y \cos ^{2} x=\cos ^{4} x$ completely by demonstrating all the steps involved.
4. Find all possible solutions of the differential equation:

$$
\begin{equation*}
y^{2} \log y=x y \frac{d y}{d x}+\left(\frac{d y}{d x}\right)^{2} \tag{15M}
\end{equation*}
$$

5. Find the orthogonal trajectories of the family of confocal conics $\frac{x^{2}}{a^{2}+\lambda}+\frac{y^{2}}{b^{2}+\lambda}=1$;
$a>b>0$ are constants and $\lambda$ is a parameter. Show that the given family of curves is self-orthogonal
6. Find the general solution of the differential equation:

$$
\begin{gathered}
x^{2} \frac{d^{2} y}{d x^{2}}-2 x(1+x) \frac{d y}{d x}+2(1+x) y=0 . . \text { Hence, solve the differential equation: } \\
x^{2} \frac{d^{2} y}{d x^{2}}-2 x(1+x) \frac{d y}{d x}+2(1+x) y=x^{3} \text { by the method of variation of parameters. }
\end{gathered}
$$

