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Mathematics-Optional

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ODE 2013-2019

UPSC – MATHEMATICS optional – 2013 Questions

1. y is a function of x, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x+y)+\sin(x+y)$. Find out a relation between x and y, which is free from any derivative/differential. **[10M]**

2. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^2 = a\sin n\theta, (r,\theta)$ being the plane polar coordinates. **[10M]**

3. Solve the differential equation $(5x^3+12x^2+6y^2)dx+6xydy=0$. **[10M]**

4. Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2}+a^2y=\sec ax$. **[10M]**

5. Find the general solution of the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\ln x\sin(\ln x)$ **[15M]**

6. By using Laplace transform method, solve the differential equation $(D^2+n^2)x = a\sin(nt+\alpha)$, $D^2=\frac{d^2}{dt^2}$ subject to the initial conditions $x=0$ and $\frac{dx}{dt}=0, at\ t=0$, in which a, n and α are constants. **[15M]**

UPSC – MATHEMATICS optional – 2014 Questions

1. Justify that a differential equation of the form:

$[y+x\,f(x^2+y^2)]dx+\ [y+y\,f(x^2+y^2)-x]dy=0$, where $f(x^2+y^2)$ is an arbitrary function of (x^2+y^2), is not an exact differential equation and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2+y^2)=(x^2+y^2)^2$. **[10M]**

2. Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency. **[10M]**

3. Solve by the method of variation of parameters. $\frac{dy}{dx}-5y=\sin x$ **[10M]**

4. Solve the differential equation $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^3y}{dx^3}+x\frac{dy}{dx}+8y=65\cos(\log_e x)$ **[20M]**

5. Solve the following differential equation: $x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x}$

When e^x is a solution to its corresponding homogeneous differential equation. **[15M]**

6. Find the sufficient condition for the differential equation $\mathrm{M}(\mathrm{x},\mathrm{y})\mathrm{dx} + \mathrm{N}(\mathrm{x},\mathrm{y})\mathrm{dy} = 0$ to have an integrating factor as a function of $(x + y)$. What will be the integrating factor in that case? Hence find the integrating factor for the differential equation $(x^2 + xy)dx + (y^2 + xy)dy = 0$, and solve it. **[15M]**

7. Solve the initial value problem $\frac{d^2y}{dt^2} + y = 8\,e^{-2t}\sin t,\; y(0) = 0, y'(0) = 0$ by using Laplace-transform. **[20M]**

UPSC – MATHEMATICS optional – 2015 Questions

1. Solve the differential equation: $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1.$ **[10M]**

2. Solve the differential equation:

$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0.$ **[10M]**

3. Find the angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$. **[10M]**

4. Find the constant a so that $(x + y)^a$ is the integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential equation. **[12M]**

5. Find the value of λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda + 2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at $(1, -1, 2)$ **[12M]**

6. (i) Obtain Laplace inverse transform of $\left\{\ln\left(1 + \frac{1}{s^2}\right) + \frac{s}{s^2+25}e^{-Rs}\right\}$

   (ii) Using Laplace transform, solve $y'' + y = t,\; y(0) = 1, y'(0) = -2.$ **[12M]**

7. Solve the differential equation $x = py - p^2$ where $p = \frac{dy}{dx}$ **[13M]**

8. Solve $x^4\frac{d^4y}{dx^4} + 6x^3\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\cos(\log_e x).$ **[13M]**

UPSC – MATHEMATICS optional – 2016 Questions

1. Find a particular integral of $\frac{d^2y}{dx^2} + y = e^{x/2}\sin\frac{x\sqrt{3}}{2}$ **[10M]**

2. Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal **[10M]**

3. Solve $\{y(1 - x\tan x) + x^2\cos x\,dx - xdy = 0\}$ **[10M]**

4. Using the method of variation of parameters, solve the differential equation $(D^2 + 2D + 1)y = e^{-x}\log(x),\; \left[D = \frac{d}{dx}\right]$ **[15M]**

5. Find the general solution of the equation $x^2\frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$. **[15M]**

6. Using Laplace transformation, solve the following: $y'' - 2y' - 8y = 0,\ y(0) = 3,$ $y'(0) = 6$ **[10M]**

UPSC – MATHEMATICS optional – 2017 Questions

1. Find the differential equation representing all the circles in the x-y plane. **[10M]**

2. Suppose that the streamlines of the fluid flow are given by a family of curves $xy = c$. Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines. **[10M]**

3. Solve the following simultaneous linear differential equations:

   $(D + 1)y = z + e^x$ and $(D + 1)z + y + e^x$ where y and z are functions of independent variable x and $D \equiv \frac{d}{dx}$. **[08M]**

4. If the growth rate of the population of bacteria at any time t is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks? **[08M]**

5. Consider the differential equation $xy\,p^2 - (x^2 + y^2 - 1)p + xy = 0$ where $p = \frac{dy}{dx}$. Substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p' = \frac{dv}{du}$. Hence, or otherwise solve the equation. **[10M]**

6. Solve the following initial value differential equations:

   $20y'' + 4y' + y = 0, y(0) = 3.2\ and\ y'(0) = 0.$ **[07M]**

7. Solve the differential equation: $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3\sin(x^2)$ **[09M]**

8. Solve the following differential equation using method of variation of parameters:

   $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 44 - 76x - 48x^2$. **[08M]**

9. Solve the following initial value problem using Laplace transform: **[17M]**

   $\frac{d^2y}{dx^2} + 9y = r(x),\ y(0) = 0,\ y'(0) = 4$ where $r(x) = \begin{cases} 8\sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x \geq \pi \end{cases}$

UPSC – MATHEMATICS optional – 2018 Questions

1. Find the angle between the tangent at a general point of the curve whose equations are $x = 3t, y = 3t^2,\ z = 3t^3$ and the line $y = z - x = 0$, **[10M]**

2. Solve $y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$ **[10M]**

3. (i) Find the Laplace transform of $f(t) = \frac{1}{\sqrt{t}}$.

(ii) Find the inverse Laplace transform of $\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}$ **[10M]**

4. Solve $y'' - y = x^2e^{2x}$ **[10M]**

5. Solve $\left(\frac{dy}{dx}\right)^2 y + 2\frac{dy}{dx}x - y = 0$ **[13M]**

6. Solve $y'' + 16y = 32\sec 2x$ **[13M]**

7. Solve $(1+x)^2y'' + (1+x)y' + y = 4\cos(\log(1+x))$ **[13M]**

8. Solve the initial value problem **[13M]**

$$y'' - 5y' + 4y = e^{2t}$$
$$y(0) = \frac{19}{12},\ y'(0) = \frac{8}{3}$$

9. Find α and β such that $x^\alpha y^\beta$ is an integrating factor of $(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ and solve the equation. **[12M]**

10. Find $f(y)$ such that $(2xe^y + 3y^2)dy + (3x^2 + f(y))dx = 0$ is exact and hence solve.

[12M]

UPSC – MATHEMATICS optional – 2019 Questions

1. Solve the differential equation

$$(2y\sin x + 3y^4 \sin x \cos x\, dx) - (4y^3\cos^2 x + \cos x)dy = 0$$ **[10M]**

2. Determine the complete solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x}\sin 2x$$ **[10M]**

3. Solve the differential equation

$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$$ **[10M]**

4. Find the Laplace transforms of $t^{-1/2}$ and $t^{1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n \in N$, is $\frac{\Gamma\left[n+1+\frac{1}{2}\right]}{s^{n+1+\frac{1}{2}}}$ **[10M]**

5. Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2y'' - 2xy' + 2y = x^3 \sin x$ and then find the general solution of the given equation by the method of variation of parameters. **[15M]**

6. Obtain the singular solution of the differential equation

$\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2 \cot^2\alpha - 2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 \operatorname{cosec}^2\alpha = 1$ Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solution. **[15M]**