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Mathematics-Optional

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Ordinary Differential Equations 2013 - 2021

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UPSC – MATHEMATICS optional – 2013 Questions

- 1. y is a function of x, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x + y) + \sin(x + y)$. Find out a relation between x and y, which is free from any derivative/differential. [10M]
- 2. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^2 = a \sin n\theta$, (r, θ) being the plane polar coordinates. [10M]
- 3. Solve the differential equation $(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0.$ [10M]
- 4. Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} + a^2y = \sec ax.$ [10M]
- 5. Find the general solution of the equation $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \ln x \sin(\ln x)$ [15M]
- 6. By using Laplace transform method, solve the differential equation $(D^2 + n^2)x = a \sin(nt + \alpha), \quad D^2 = \frac{d^2}{dt^2}$ subject to the initial conditions x = 0 and $\frac{dx}{dt} = 0$, at t = 0, in which a, n and α are constants. [15M]

UPSC – MATHEMATICS optional – 2014 Questions

1. Justify that a differential equation of the form:

 $[y + x f(x^{2} + y^{2})]dx + [y + y f(x^{2} + y^{2}) - x]dy = 0, \text{ where } f(x^{2} + y^{2}) \text{ is an}$ arbitrary function of $(x^{2} + y^{2})$, is not an exact differential equation and $\frac{1}{x^{2} + y^{2}}$ is an integrating factor for it. Hence solve this differential equation for $f(x^{2} + y^{2}) = (x^{2} + y^{2})^{2}.$ [10M]

- Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency. [10M]
- **3**. Solve by the method of variation of parameters. $\frac{dy}{dx} 5y = \sin x$ [10M]
- **4**. Solve the differential equation $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos(\log_e x)$ [20M]
- 5. Solve the following differential equation: $x \frac{d^2y}{dx^2} 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$

When e^x is a solution to its corresponding homogeneous differential equation. [15M]

6. Find the sufficient condition for the differential equation M(x, y)dx + N(x, y)dy = 0 to have an integrating factor as a function of (x + y). What will be the integrating factor in that case? Hence find the integrating factor for the differential equation $(x^2 + xy)dx + (y^2 + xy)dy = 0$, and solve it. [15M]

7. Solve the initial value problem $\frac{d^2y}{dt^2} + y = 8 e^{-2t} \sin t$, y(0) = 0, y'(0) = 0 by using Laplace-transform. [20M]

UPSC – MATHEMATICS optional – 2015 Questions

- **1.** Solve the differential equation: $x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1.$ [10M]
- **2**. Solve the differential equation:

$$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0.$$
 [10M]

- **3**. Find the angle between the surfaces $x^2 + y^2 + z^2 9 = 0$ and $z = x^2 + y^2 3$ at (2, -1, 2). [10M]
- 4. Find the constant a so that $(x + y)^a$ is the integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential equation. [12M]
- 5. Find the value of λ and μ so that the surfaces $\lambda x^2 \mu yz = (\lambda + 2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at (1, -1, 2) [12M]
- **6**. (i) Obtain Laplace inverse transform of $\left\{ \ln\left(1 + \frac{1}{s^2}\right) + \frac{s}{s^2 + 25}e^{-Rs} \right\}$

(ii) Using Laplace transform, solve
$$y'' + y = t$$
, $y(0) = 1$, $y'(0) = -2$. [12M]

- 7. Solve the differential equation $x = py p^2$ where $p = \frac{dy}{dx}$ [13M]
- 8. Solve $x^4 \frac{d^4 y}{dx^4} + 6x^3 \frac{d^3 y}{dx^3} + 4x^2 \frac{d^2 y}{dx^2} 2x \frac{dy}{dx} 4y = x^2 + 2\cos(\log_e x).$ [13M]

UPSC – MATHEMATICS optional – 2016 Questions

- **1.** Find a particular integral of $\frac{d^2y}{dx^2} + y = e^{x/2} \sin \frac{x\sqrt{3}}{2}$ [10M]
- **2.** Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal [10M]
- 3. Solve $\{y(1 x \tan x) + x^2 \cos x \, dx x \, dy = 0\}$ [10M]
- 4. Using the method of variation of parameters, solve the differential equation $(D^2 + 2D + 1)y = e^{-x} \log(x), \left[D = \frac{d}{dx}\right]$ [15M]

- 5. Find the general solution of the equation $x^2 \frac{d^3y}{dx^3} 4x \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} = 4.$ [15M]
- 6. Using Laplace transformation, solve the following: y'' 2y' 8y = 0, y(0) = 3, y'(0) = 6 [10M]

UPSC – MATHEMATICS optional – 2017 Questions

- **1.** Find the differential equation representing all the circles in the *x*-*y* plane. [10M]
- 2. Suppose that the streamlines of the fluid flow are given by a family of curves xy = c. Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines. [10M]
- 3. Solve the following simultaneous linear differential equations:

 $(D + 1)y = z + e^x$ and $(D + 1)z + y + e^x$ where y and z are functions of independent variable x and $D \equiv \frac{d}{dx}$. [08M]

- 4. If the growth rate of the population of bacteria at any time *t* is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks? [08M]
- 5. Consider the differential equation $xy p^2 (x^2 + y^2 1)p + xy = 0$ where $p = \frac{dy}{dx}$. Substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p' = \frac{dv}{du}$. Hence, or otherwise solve the equation. [10M]
- 6. Solve the following initial value differential equations:

$$20y'' + 4y' + y = 0, y(0) = 3.2 \text{ and } y'(0) = 0.$$
 [07M]

- 7. Solve the differential equation: $x \frac{d^2 y}{dx^2} \frac{dy}{dx} 4x^3 y = 8x^3 \sin(x^2)$ [09M]
- 8. Solve the following differential equation using method of variation of parameters:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 44 - 76x - 48x^2.$$
 [08M]

9. Solve the following initial value problem using Laplace transform: [17M]

$$\frac{d^2y}{dx^2} + 9y = r(x), \ y(0) = 0, \ y'(0) = 4 \text{ where } r(x) = \begin{cases} 8\sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x \ge \pi \end{cases}$$

UPSC – MATHEMATICS optional – 2018 Questions

- 1. Find the angle between the tangent at a general point of the curve whose equations are x = 3t, $y = 3t^2$, $z = 3t^3$ and the line y = z x = 0, [10M]
- **2.** Solve $y''' 6y'' + 12y' 8y = 12e^{2x} + 27e^{-x}$ [10M]
- **3.** (i) Find the Laplace transform of $f(t) = \frac{1}{\sqrt{t}}$.

(ii) Find the inverse Laplace transform of
$$\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}$$
 [10M]

4. Solve
$$y'' - y = x^2 e^{2x}$$
 [10M]

- 5. Solve $\left(\frac{dy}{dx}\right)^2 y + 2\frac{dy}{dx}x y = 0$ [13M]
- 6. Solve $y'' + 16y = 32 \sec 2x$ [13M]

7. Solve
$$(1+x)^2 y'' + (1+x)y' + y = 4\cos(\log(1+x))$$
 [13M]

8. Solve the initial value problem

$$y'' - 5y' + 4y = e^{2t}$$

$$y(0) = \frac{19}{12}, \ y'(0) = \frac{8}{3}$$

- 9. Find α and β such that $x^{\alpha}y^{\beta}$ is an integrating factor of $(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ and solve the equation. [12M]
- 10. Find f(y) such that $(2xe^y + 3y^2)dy + (3x^2 + f(y))dx = 0$ is exact and hence solve.

[12M]

[13M]

UPSC – MATHEMATICS optional – 2019 Questions

1. Solve the differential equation

$$(2y\sin x + 3y^4\sin x\cos x\,dx) - (4y^3\cos^2 x + \cos x)dy = 0$$
 [10M]

2. Determine the complete solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2 e^{2x} \sin 2x$$
[10M]

3. Solve the differential equation

$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$$
[10M]

- 4. Find the Laplace transforms of $t^{-1/2}$ and $t^{1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2'}}$ where $n \in N$, is $\frac{\Gamma[n+1+\frac{1}{2}]}{s^{n+1+\frac{1}{2}}}$ [10M]
- 5. Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2y'' 2xy' + 2y = x^3 \sin x$ and then find the general solution of the given equation by the method of variation of parameters. [15M]
- 6. Obtain the singular solution of the differential equation

 $\left(\frac{dy}{dx}\right)^2 \left(\frac{y}{x}\right)^2 \cot^2 \alpha - 2\left(\frac{dy}{dx}\right) \left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 \csc^2 \alpha = 1$ Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solution. [15M]

UPSC – MATHEMATICS optional – 2020 Questions

1. Solve the following differential equation:

$$x\cos\left(\frac{y}{x}\right)(y\,dx + x\,dy) = y\sin\left(\frac{y}{x}\right)(x\,dy - y\,dx)$$
[10M]

- **2.** Find the orthogonal trajectories of the family of circles passing through the points (0, 2) and (0, -2). **[10M]**
- 3. Using the method of variation of parameters, solve the differential equation

$$y'' + (1 - \cot x)y' - y \cot x = \sin^2 x$$
, if $y = e^{-x}$ is one solution of CF. [20M]

4. Using Laplace transform, solve the initial value problem ty'' + 2ty' + 2y = 2;

$$y(0) = 1$$
 and $y'(0)$ is arbitrary. Does this problem have a unique solution? [10M]

5. Solve the following differential equation:

$$(x+1)^2 y'' - 4(x+1)y' + 6y = 6(x+1)^2 + \sin\log(x+1)$$
 [10M]

6. Find the general and singular solutions of the differential equation

$$9p^2(2-y)^2 = 4(3-y)$$
, where $p = \frac{dy}{dx}$. [10M]

UPSC – MATHEMATICS optional – 2021 Questions

- **1.** Solve the differential equation: $\frac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x \cos 2x$ [10M]
- 2. Solve the initial value problem: $\frac{d^2y}{dx^2} + 4y = e^{-2x} \sin 2x$; y(0) = y'(0) = 0 using Laplace transform method. [10M]

- 3. Solve the equation: $\frac{d^2y}{dx^2} + (\tan x 3\cos x)\frac{dy}{dx} + 2y\cos^2 x = \cos^4 x$ completely by demonstrating all the steps involved. [15M]
- 4. Find all possible solutions of the differential equation:

$$y^{2}\log y = xy\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{2}$$
[15M]

5. Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$; a > b > 0 are constants and λ is a parameter. Show that the given family of curves is self-orthogonal [10M]

6. Find the general solution of the differential equation:

[10M]

 $x^{2} \frac{d^{2}y}{dx^{2}} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = 0$. Hence, solve the differential equation: $x^{2} \frac{d^{2}y}{dx^{2}} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^{3}$ by the method of variation of parameters.