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Mathematics-Optional

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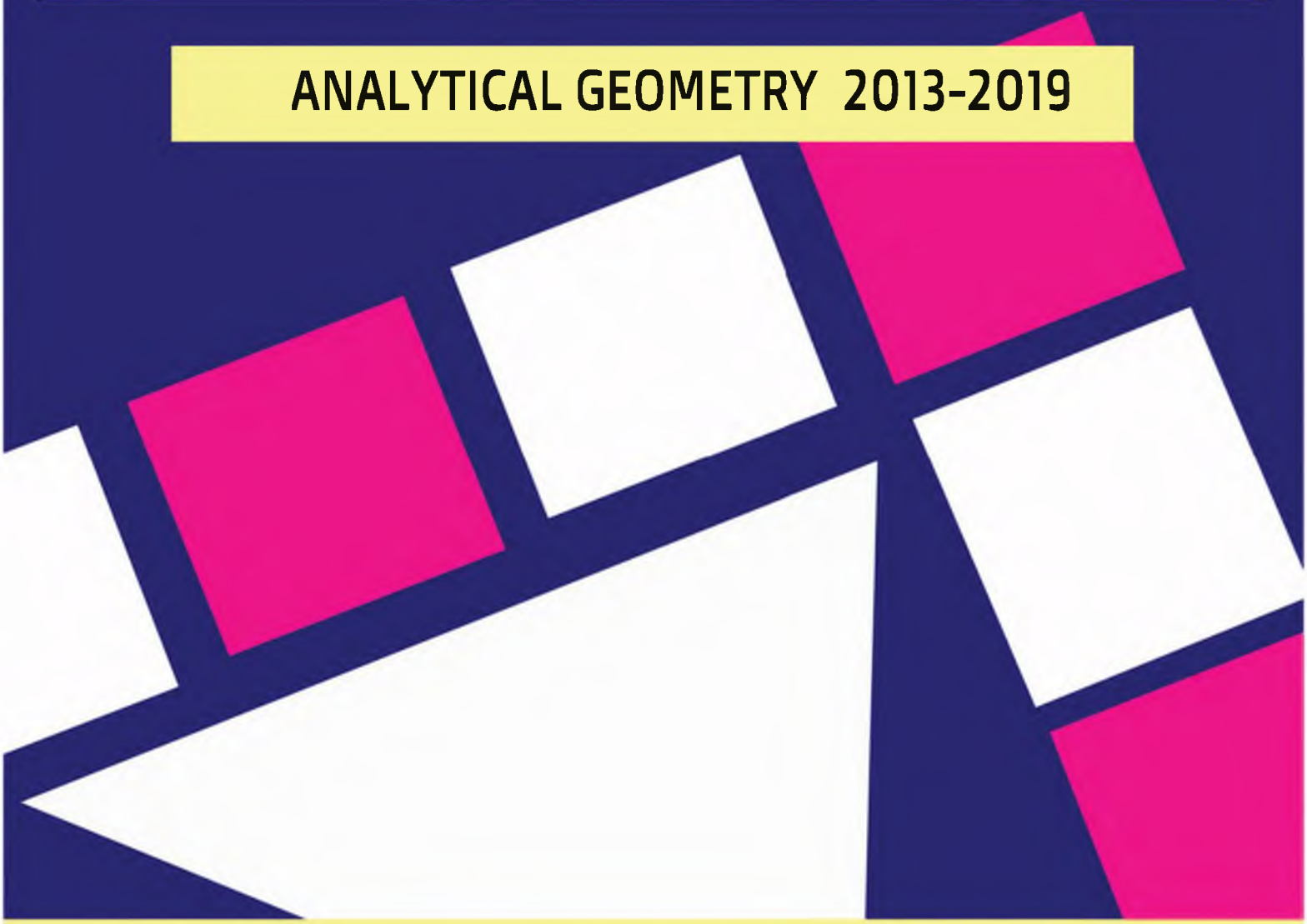
ANALYTICAL GEOMETRY 2013-2019



UPSC – MATHEMATICS optional – 2013 Questions

1. Find the equation of the plane which passes through the points $(0, 1, 1)$ and $(2, 0, -1)$ and is parallel to the line joining the points $(-1, 1, -2), (3, -2, 4)$. Find also the distance between the line and the plane. **[10M]**

2. A sphere S has points $(0, 1, 0), (3, -5, 2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. **[10M]**

3. Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $2(x^2 + y^2 + z^2) = 3r^2$ **[15M]**

4. A cone has for its guiding curve the circle $x^2 + y^2 + 2ax + 2by = 0, z = 0$ and passes through a fixed point $(0, 0, c)$. If the section of the cone by the plane $y = 0$ is a rectangular hyperbola, prove that the vertex lies on the fixed circle.

$$x^2 + y^2 + z^2 + 2ax + 2by = 0,$$
$$2ax + 2by + cz = 0.$$

[15M]

5. A variable generator meets two generators of the system through the extremities B and B' of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - z^2c^2 = 1$ in P and P'. Prove that $BP \cdot B'P' = a^2 + c^2$. **[20M]**

UPSC – MATHEMATICS optional – 2014 Questions

1. Find the co-ordinates of the points on the sphere $x^2 + y^2 + z^2 - 4x + 2y = 4$, the tangent planes at which are parallel to the plane $2x - y + 2z = 1$. **[10M]**

2. Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$, represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$. **[10M]**

3. Show that the lines drawn from the origin parallel to the normal to the central conicoid $ax^2 + by^2 + cz^2 = 1$, at its points of intersection with the $p^2\left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c}\right)^2$. **[15M]**

4. Find the equations of the two generating lines through any point $(a\cos\theta,\ b\sin\theta,\ 0)$, of the principal elliptic section $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$, of the hyperboloid by the plane $z = 0$. **[15M]**

5. . Examine whether the plane $x + y + z = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines. **[10M]**

UPSC – MATHEMATICS optional – 2015 Questions

1. For what positive value of a, the plane $ax - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and hence find the point of contact. **[10M]**

2. If $6x = 3y = 2z$ represents one of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then obtain the equations of the other two generators. **[13M]**

3. Which point of the sphere $x^2 + y^2 + z^2 = 1$ is at the maximum distance from the point $(2, 1, 3)$? **[13M]**

4. Obtain the equation of the plane passing through the points $(2, 3, 1)$ and $(4, -5, 3)$ parallel to x-axis. **[06M]**

5. Verify if the lines: $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar. If yes, then find the equation of the plane in which they lie. **[07M]**

6. Two perpendicular tangent planes to the paraboloid $x^2 + y^2 = 2z$ intersect in a straight line in the plane $x = 0$. Obtain the curve to which this straight line touches. **[13M]**

UPSC – MATHEMATICS optional – 2016 Questions

1. Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4; z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3. **[10M]**

2. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z - 3$ and $y - mx = z = 0$. For what value of m will the two lines intersect? **[10M]**

3. Find the surface generated by a line which intersects the lines $y = a = z, x + 3z = a = y + z$ and parallel to the plane $x + y = 0$. **[10M]**

4. Show that the cone $3yz - 2zx - 2xy = 0$ has an infinite set of three mutually perpendicular generators. If $\frac{x}{1} = \frac{y}{1} = \frac{z}{2}$ is a generator belonging to one such set, find the other two. **[10M]**

5. Find the locus of the point of intersection of three mutually perpendicular tangent planes to the conicoid $ax^2 + by^2 + cz^2 = 1$. **[10M]**

UPSC – MATHEMATICS optional – 2017 Questions

1. Find the equation of the tangent plane at point (1, 1, 1) to the conicoid $3x^2 - y^2 = 2z$. **[10M]**

2. Find the shortest distance between the skew lines: $\frac{x-3}{3} = \frac{8-y}{1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ **[10M]**

3. A plane passes through a fixed point (a, b, c) and cuts the axes at the points A, B, C respectively. Find the locus of the centre of the sphere which passes through the origin O and A, B, C. **[15M]**

4. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Find the point of contact. **[10M]**

5. Find the locus of the point of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1$. **[10M]**

6. Reduce the following equation to the standard form and hence determine the nature of the conicoid : $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$. **[15M]**

UPSC – MATHEMATICS optional – 2018 Questions

7. Find the projection of the straight line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{-1}$ on the plane $x + y + 2z = 6$. **[10M]**

8. Find the equations to the generating lines of the paraboloid $(x + y + z)(2x - y - z) = 6z$ which pass through the point $(1, 1, 1)$. **[12M]**

9. Find the equation of the sphere in xyz-plane passing through the points $(0, 0, 0), (0, 1, -1)$ and $(-1, 2, 0)$ and $(1, 2, 3)$. **[13M]**

10. Find the equation of the cone with $(0, 0, 1)$ as the vertex and $2x^2 - y^2 = 4, z = 0$ as the guiding curve. **[13M]**

11. Find the equation of the plane parallel to $3x - y + 3z = 8$ and passing through the point $(1, 1, 1)$. **[12M]**

UPSC – MATHEMATICS optional – 2019 Questions

1. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ intersect. Find the coordinates of the point of intersection and the equation of the plane containing them. **[10M]**

2. The plane $x + 2y + 3z = 12$ cuts the axes of coordinates in A, B, C. Find the equations of the circle circumscribing the triangle ABC. **[10M]**

3. Prove that the plane $z = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has the vertex at $(2, 4, 1)$ in a rectangular hyperbola. **[10M]**

4. Prove that, in general, three normal can be drawn from a given point to the paraboloid $x^2 + y^2 = 2az$, but if the point lies on the surface $27a(x^2 + y^2) + 8(a - z)^3 = 0$ then two of the three normal coincide. **[15M]**

5. Find the length of the normal chord through a point P of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and prove that if it is equal to $4\,PG^3$, where G_3 is the point where the normal chord through P meets the xy-plane, then P lies on the cone

$$\frac{x^2}{a^2}(2c^2 - a^2) + \frac{y^2}{b^6}(2c^2 - b^2) + \frac{z^2}{c^4} = 0$$ **[15M]**